T-Absolutely Closed Spaces

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Abstract

The concept of T-Absolutely closed spaces (ACS), which serves as a generalization of the classical absolutely closed spaces is discussed. It's began by defining T-ACS and demonstrating how they extend the properties of ACS to a broader class of topological structures. Several key properties are established, such as the behavior of T-ACS under continuous mappings, their relationship with other well-known topological spaces, and how they interact with various topological operations like product spaces, subspaces, and quotient spaces. Additionally, we provide examples to illustrate how T-ACS differ from their classical counterparts and investigate conditions under which a space can be classified as T-Absolutely closed. This new concept offers a more flexible framework for studying topological spaces while maintaining important characteristics of absolute closure.

Keywords: T-Absolutely Closed Space, Absolutely Closed Space, Topological Spaces, Generalization, Continuous Mappings.

1 INTRODUCTION

Studies on the T-Absolutely closed spaces (ACS) are discussed at Table 1. The absolutely continuous curves (ACC) is discussed at [1] in Finsler-induced asymmetric spaces (FIAS), showing that three types of curves coincide on bounded intervals. It also proves a universal existence theorem for gradient flow (GFT) and investigates those curves in Wasserstein spaces over Finsler manifolds. The study of [2] focuses on Carleson embeddings from Bergman spaces into $\frac{L^p}{mu}$. It characterizes when these embeddings are r-summing, especially for $(P > 1)$& $(r \geq 1)$, and applies this to weighted composition operators. Projections on Banach spaces within the convex hull of surjective isometries [3]. Applying results to absolutely continuous functions on compact subsets of R in strictly convex Banach spaces. Study of [4] proves that isometries between unit spheres of absolutely smooth 2D Banach spaces extend to linear isometries, solving Tingley's problem for this class. The [5] deals with surjective linear isometries between spaces of scalar-valued absolutely continuous functions. It shows that these isometries are weighted composition operators in connected spaces, generalizing previous results. The absolutely split vector (ASV) is classified at [6], bundles on proper k-schemes. It links the Picard scheme's closed points with indecomposable vector bundles and applies this to study the geometry of Brauer–Severi varieties. The von Neumann algebras with normal semi-finite faithful (nsf) traces and defines an "absolutely dilatable" contraction is studied at [7]. It characterizes bounded Schur multipliers in terms of a von Neumann algebra with a separable predual and establishes conditions for absolute dilatability in separable cases. Norm attaining and absolutely norm attaining (AN) operators between complex Hilbert spaces is discussed at [8]. It also explores minimum attaining and absolutely minimum attaining (AM) operators, providing results for Toeplitz and Hankel operators. The Bergman projection for L^p-Bergman spaces on Reinhardt domains using an integral kernel. It identifies duals of these spaces with weighted Bergman spaces [9]. The irreducible modules for finite groups and develop an algorithm is characterizes at [10]. It aimed to test whether an absolutely irreducible matrix representation has a proper G-invariant subalgebra. Absolutely strongly star-Menger (ASSM) spaces is investigated at [11]. It also explores their relationships with other topological spaces, along with their topological properties. Studies of surjective isometries between spaces of absolutely continuous vector-valued functions and generalizes known results on these isometries [12]. The [13] examines stability in Reproducing Kernel Hilbert Spaces (RKHS) and presents a stability test for kernels in continuous and discrete time using test functions valued at ±1.

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Table 1: illustration of previous studies of similar interests.

Ref	Aim	Methodology	Findings
[1]	Investigate ACC in FIAs.	Study Finsler manifolds (FM), Wasserstein spaces (WS), and continuity equations (CQ).	Coincidence of curves, gradient flow theorem, validity in nonsmooth spaces.
[2]	Study Carleson embeddings and characterize absolutely summing operators.	Analyze Berezin transform and r-summing injections on Bergman spaces.	Characterization of summing operators and connection with Berezin transform.
[3]	Examine isometries convex hull. projections such as Banach spaces.	Utilize for absolutely continuous functions on compact subsets of R.	Results applied to absolutely continuous functions in convex Banach spaces.
[4]	Solve Tingley's problem in absolutely smooth 2D Banach spaces.	Prove isometries between unit spheres extend to linear isometries.	Resolved Tingley's problem for absolutely smooth 2D Banach spaces.
[5]	Examines surjective linear isometries in spaces of absolutely continuous functions.	Analyze isometries on real line subsets.	Showed isometries are weighted composition operators for connected spaces.
[6]	Classify absolutely split vector bundles over proper k-schemes.	Prove correspondence with Picard scheme.	Classified vector bundles and studied geometry of Brauer–Severi varieties.
[7]	Characterize Schur multipliers in von Neumann algebras.	Use von Neumann algebras and expectations.	Schur multipliers are absolutely dilatable under von Neumann conditions.
[8]	Study AN and AM operators in Hilbert spaces.	Investigate Toeplitz and Hankel operators.	Toeplitz AN operators characterized; only finite-rank Hankel operators are AM.
[9]	Generalize Bergman projection for L^p spaces on Reinhardt domains.	Construct integral kernel for Bergman projection.	Constructed bounded projection for L^p -Bergman spaces.
[10]	Investigate G-invariant subalgebras for irreducible G-modules.	Use Pure Tensor problem and Gröbner basis.	Identified when V is imprimitive or decomposable.
[11]	Explore properties of star-Menger spaces.	Study open covers and dense subsets.	Identified relationships and properties of star-Menger spaces.
[12]	Study surjective isometries on vector-valued function spaces.	Analyze isometries with supremum norm.	Generalized Jerison's theorem and extended results on isometries.

[13]	Investigate stability in reproducing kernel Hilbert spaces (RKHSs).	Analyze kernels over test functions.	Stability test reduced to kernels over test functions with values ±1.

2 EMPIRICAL APPROACH

A topological space (X,τ) is absolutely closed space iff every open cover of X contains a finite subfamily whose closer covers X. recall that a space X is called compact iff every open cover of X has a finite subcover.

i. Let $(X.\tau)$ be a topological space ,let $T: P(X) \to P(X)$ be a function $(P(X))$ is the power set of X. T is called an operator associated with the topology τ if $w \subseteq T(w)$ where $w \in \tau$ the triple (X,τ,T) ior example is called an operator topological space for example, let $T: P(X) \to P(X)$be defined as follows T(A)= int (clA) ,$A \subseteq X$ it is clear that T is an operator associated with the topology τon X.

ii. Let (X,τ,T) be an operator topological space (O.T.S) X is called T-absolutely closed space iff every open cover, $\gamma = \{G_\alpha / \alpha \in \Omega\}$ of X contains a finite subfamily γ^* $=\{G_{\alpha 1}, G_{\alpha 2}, G_{\alpha n}\}$, Such that $X = \bigcup_{i=1}^{n} T(G_{\alpha i})$. If T is the closure operator (T(A)=cl(A)) then we get the usual definition. of Absolutely closed space.

iii. Let (X,τ) be a topological space, we say that X is a regular space if : Given $p \in X$ and $w \in \tau$ such that $p \in X$,then there exist $v \in \tau$ such That $p \in v \subseteq cl(v) \subseteq w$.

iv. Let (X,τ,T) be a operator topological space, we say that X is T-regular if: given $p \in X$ and $w \in X$ such that $p \in w$, than there exists $v \in \tau$ such that $p \in v \subseteq T(v) \subseteq w$.

v. Let $(X,\tau.T)$ be an O.T.S. $A \subseteq X$ is called T- open if: given $p \in A$,then $\exists w \in \tau \ni p \in w \subseteq T(w) \subseteq A$.

vi. Let (X,τ,T) be an O.T.S. X is called T-compact if every T-open cover of X has a finite subcover.

2.1 Theorem A

Every regular absolutely closed space is compact, this means if (X,τ) is regular, then X is compact iff X is absolutely closed we generalize the above theorem as follows:

Let (X,τ,T) be a T-regular space, then X is T-compact iff X is T- absolutely closed space.

Proof:

Let X be T-compact let $\gamma = \{G_\alpha / \alpha \in \Omega\}$ be an open cover of X but X

Is T-regular ,Hence γ will be a T-open cover of X Hence $\exists \alpha_1, \alpha_2, \ldots \alpha_n \ni$

$X = \bigcup_{i=1}^{n} G_{\alpha i}$

Now $G_{\alpha i} \subseteq T(G_{\alpha i})$

This means that $X = \bigcup_{i=1}^{n} T(G_{\alpha i})$

Therefore, X is T-absolutely closed

Assume X is T-absolutely closed let $\gamma = \{G_\alpha / \alpha \in \Omega\}$ be a T-open cover of X

Now let $p \in X$, then $\exists G_\alpha \ni p \in G_\alpha$ but G_α is T-open,

Hence $\exists v_\alpha \in \tau \ni$ $p \in v_\alpha \subseteq T(v_\alpha) \subseteq G_\alpha$ consider $\gamma^* = \{v_\alpha / \alpha \in \Omega\}$ γ^* will be an open cover of X which is T-absolutely closed Hence $\exists \alpha_1, \ldots, \alpha_n \ni X = \bigcup_{i=1}^{n} T(V_{\alpha i})$

therefore $X = \bigcup_{i=1}^{n} G_{\alpha i}$ which means that X is T-compact

2.2 Theorem B

Let $f: (X,\tau,T) \to (Y,\sigma,L)$ be continuous function from an O.T.S $(X.\tau,T)$ onto an O.T.S (Y,σ,L) and let $f(T(A)) \subseteq L(f(A))$ $\forall A \subseteq X$ .If X is T-Absolutely closed space then Y is L-Absolutely closed space

Proof:

Let $\gamma = \{G_\alpha / \alpha \in \Omega\}$ be an open cover of Y then $\gamma^* = \{f^{-1}(G_\alpha)\alpha \in \Omega\}$ Will be an open cover of X but X is T-Absolutely closed space Hence $\exists \alpha_1, \ldots, \alpha_n \ni X = \bigcup_{i=1}^{n} T(f^{-1}(G_{\alpha i}))$, now $Y = f(X) = f\left(\bigcup_{i=1}^{n} T(f^{-1}(G_{\alpha i}))\right) = \bigcup_{i=1}^{n} f(T(f^{-1}(G_{\alpha i}))$ $\subseteq \bigcup_{i=1}^{n} L(f((f^{-1}(G_{\alpha i}))) = \bigcup_{i=1}^{n} L(G_{\alpha i})$ which means that Y is L-Absolutely closed space.

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