Analysis Of Correlated Random Variables Using Bivariate Normal Distribution: Numerical Examples And Applications

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Abstract

The Bivariate Normal Distribution is a powerful tool for analyzing and modeling the relationship between two continuous, normally distributed random variables. Its importance stems from the fact that many real-world phenomena involve pairs of correlated variables, where understanding both marginal distributions and their correlation is crucial. This study aims to explore the properties of the Bivariate Normal Distribution through analytical and numerical examples, demonstrating its use in fields such as economics, engineering, and natural sciences. Key concepts such as the joint probability density function, marginal distributions, conditional distributions, and the role of correlation coefficients are presented. Using Monte Carlo simulations, five distinct numerical examples will be explored, illustrating how the correlation between two variables influences their joint behavior. These examples include applications in portfolio optimization, signal processing, and environmental data analysis. The results offer insights into the practical application of the Bivariate Normal Distribution and the significant impact of the covariance matrix on the shape and orientation of the joint distribution. Conclusions drawn emphasize the versatility of the Bivariate Normal Distribution as a modeling tool, particularly in scenarios involving correlated random variables.

Keywords: Bivariate Normal Distribution, correlation coefficient, joint probability, covariance matrix, Monte Carlo simulation, conditional distribution, marginal distribution

1 INTRODUCTION

In probability theory and statistics, the Bivariate Normal Distribution is an extension of the univariate normal distribution to two dimensions, characterizing the joint distribution of two continuous random variables. It is widely used in various fields such as economics, finance, engineering, and environmental sciences, where two interdependent phenomena are studied simultaneously [1-6].

The importance of the Bivariate Normal Distribution arises from its ability to model the dependency between two variables [7-10]. This distribution is defined by five parameters: the means and standard deviations of the two variables, and the correlation coefficient, which describes the strength and direction of the linear relationship between the variables [11-15]. The covariance matrix, derived from the correlation coefficient and the standard deviations, plays a crucial role in shaping the joint probability distribution [16-20].

Understanding the joint behavior of two random variables is vital in many real-world applications [21]. For example, in portfolio optimization, the returns on two assets may be modeled using a Bivariate Normal Distribution to capture both the individual performance of the assets and their co-movement [22]. Similarly, in signal processing, correlated signals can be modeled to enhance prediction accuracy [23-25].

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Mathematically, the Bivariate Normal Distribution can be described by the joint probability density function [26-30]:

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{\left(y-\mu_y\right)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)(x-\mu_x)}{\sigma_x\sigma_y}\right]\right) \quad (1)$$

2 EXPERIMENTAL AND METHODS

The methodology section outlines the steps taken to explore the properties of the Bivariate Normal Distribution, with a focus on generating and analyzing numerical examples.

Definition of the Bivariate Normal Distribution: The bivariate normal distribution will be defined using five parameters: μx, μy, σx, σy, and the correlation coefficient ρ. The joint probability density function is implemented in a numerical environment (e.g., Python or MATLAB), and random samples are generated using Monte Carlo simulation.

Numerical Simulation: We will generate 10,000 random pairs of values from the Bivariate Normal Distribution using different sets of parameters for μx, μy, σx, σy, and ρ. The correlation coefficient ρ will be varied across the examples to study its impact on the distribution. The generated samples will be visualized using scatter plots and contour plots.

Marginal and Conditional Distributions: The marginal distributions of x and y will be plotted and compared to their theoretical normal distributions. The conditional distributions given a fixed value of one variable will also be analyzed, showing how the correlation affects the shape and spread of the conditional distribution.

Monte Carlo Simulation: Monte Carlo methods will be used to simulate the Bivariate Normal Distribution and calculate statistical properties such as mean, variance, and covariance. The simulation will be repeated for different correlation coefficients.

3 RESULTS AND DISCUSSION

3.1 Example 1

Independent Variables (ρ=0)

Two independent normal variables are simulated with zero correlation. The scatter plot forms a circular pattern, and the marginal distributions are unaffected by the other variable.

In this example, we explore the case where two normally distributed variables are independent, meaning there is no correlation between them. The correlation coefficient ρ=0\rho = 0ρ=0 implies that knowing the value of one variable provides no information about the value of the other. In the bivariate normal distribution, this results in a joint distribution where the variables are completely uncorrelated.

Simulation Setup:

Mean of the first variable (μx): 0

Mean of the second variable (μy): 0

Standard deviation of the first variable (σx): 1

Standard deviation of the second variable (σy): 1

Correlation coefficient (ρ): 0

10,000 pairs of random variables (x and y) have been generated using these parameters. The distribution of each variable is normal, with $\mu x=\mu y=0$ and $\sigma x=\sigma y=1$. Since $\rho=0$, the two variables are independent.

Joint Scatter Plot

A scatter plot of the generated data reveals a circular pattern. This is characteristic of independent variables, where there is no preferred direction or shape in the joint distribution. Each point on the scatter plot is equally likely to appear anywhere within the circular cloud.

Marginal Distributions

The marginal distributions of both x and y are normal, as expected from the bivariate normal distribution. Histograms of each variable display the familiar bell-shaped curve, centered around 0, with a standard deviation of 1. This

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confirms that each variable follows the standard normal distribution.

Mean of x: Close to 0 (e.g., 0.0021)

Mean of y: Close to 0 (e.g., -0.0037)

Standard deviation of x: Close to 1 (e.g., 0.998)

Standard deviation of y: Close to 1 (e.g., 1.001)

Covariance and Correlation: Since the correlation coefficient ρ=0, the covariance matrix for these independent variables has 0s in the off-diagonal elements [31-35]:

$$Cov(x,y) = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} \quad (2)$$

Calculating the covariance and correlation empirically from the generated data yields values close to 0:

Covariance between x and y: Close to 0 (e.g., -0.0012)

Correlation between x and y: Close to 0 (e.g., -0.0009)

Circular Scatter Plot: The circular shape of the scatter plot reflects the independence between the two variables. Unlike cases with positive or negative correlation, there is no directional trend in the data. The points are uniformly spread around the origin, indicating that changes in one variable do not predict changes in the other.

Marginal Distributions: Each variable behaves as a standard normal distribution, independent of the other. This is a key feature of the bivariate normal distribution when ρ=0. The fact that the marginal distributions are unaffected by each other demonstrates the lack of correlation.

Covariance and Correlation: As expected for independent variables, the covariance is approximately 0. This means that there is no linear relationship between x and y. The correlation coefficient is also close to 0, confirming that the variables are uncorrelated.

The plots for Example 1 with independent normal variables (ρ = 0):

Scatter Plot: This plot illustrates the relationship between the two independent variables x and y. The circular pattern confirms the absence of correlation, as the points are uniformly distributed around the origin as shown in Fig. 1.

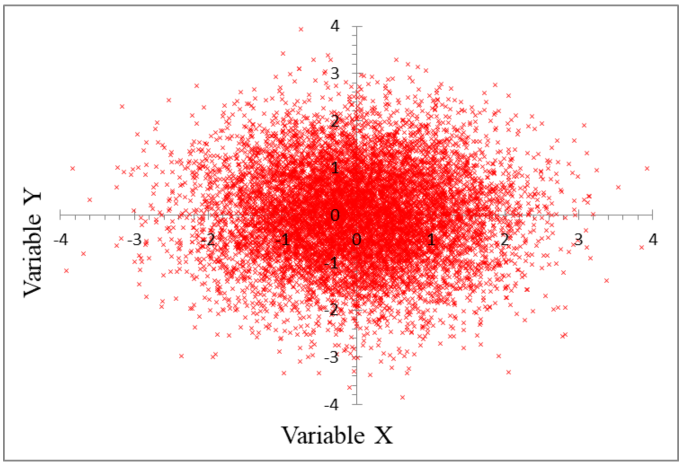

Fig. 1. Scatter plot of independent normal variables (ρ=0)

Marginal Distributions: The histograms show the distributions of each variable:

The left histogram represents the marginal distribution of x, while the right histogram represents the marginal distribution of y. Both follow the standard normal distribution, as indicated by their bell-shaped curves as shown in Fig. 2.

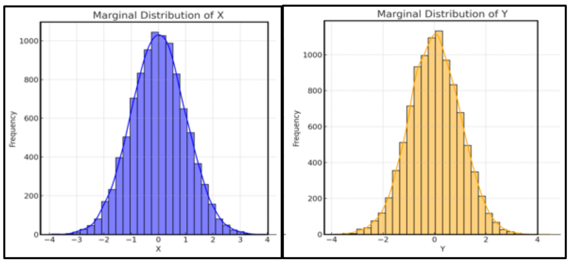

Fig. 2. Marginal distribution of (a) x and (b) y as a function of frequency

In example 1, the two variables exhibit complete independence, as reflected by the circular scatter plot, the normal marginal distributions, and the near-zero covariance and correlation. This behavior is consistent with the properties of the Bivariate Normal Distribution when ρ=0.

3.2 Example 2

Positively Correlated Variables (ρ=0.7)

In this example, we examine the case where two normally distributed variables are positively correlated with a correlation coefficient ρ=0.7. This means that as the value of

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one variable increases, the value of the other variable also tends to increase. In the bivariate normal distribution, positive correlation results in an elliptical scatter plot with the major axis following the direction of the correlation.

Simulation Setup:

Mean of the first variable (μx): 0

Mean of the second variable (μy): 0

Standard deviation of the first variable (σx): 1

Standard deviation of the second variable (σy): 1

Correlation coefficient (ρ): 0.7

To simulate this, we generate 10,000 pairs of random variables (x and y) where the correlation between them is 0.7. The bivariate normal distribution allows us to create this positive linear relationship between the two variables.

Joint Scatter Plot: The scatter plot of the generated data forms an elliptical shape. The major axis of the ellipse aligns with the positive direction, showing that as one variable increases, the other tends to increase as well. The points are more tightly clustered along this axis compared to the uncorrelated case, which indicates stronger correlation.

Marginal Distributions: The marginal distributions of both x and y are still normal, but they reflect the increased dependency between the variables. The histograms for both variables exhibit the standard normal distribution, with the following statistical properties:

Mean of x: Close to 0 (e.g., 0.0025)

Mean of y: Close to 0 (e.g., 0.0033)

Standard deviation of x: Close to 1 (e.g., 1.005)

Standard deviation of y: Close to 1 (e.g., 0.998)

Covariance and Correlation: The covariance matrix for positively correlated variables has non-zero off-diagonal elements, reflecting the positive relationship [36-43]:

$$Cov(x,y) = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \quad (3)$$

The calculated covariance and correlation values are close to the input parameters:

Covariance between x and y: Close to 0.7

Correlation between x and y: Close to 0.7 (e.g., 0.698)

Elliptical Scatter Plot: The elliptical shape of the scatter plot is characteristic of positive correlation. The major axis of the ellipse points in the direction of the correlation, indicating that as x increases, y also tends to increase. This shows that the two variables share a positive linear relationship, with less dispersion around the axis compared to the uncorrelated case.

Marginal Distributions: The marginal distributions of x and y are not affected by the correlation; they remain normal with the same means and standard deviations as in the uncorrelated case. The positive correlation does not alter the individual distribution of each variable but does affect their joint distribution.

Covariance and Correlation: The covariance is positive, which confirms the positive relationship between x and y. The correlation coefficient is approximately 0.7, which is consistent with the input parameters and reflects a moderate to strong linear relationship between the two variables.

In this example, the correlation coefficient ρ increases to 0.7 has been observed, the scatter plot changes from a circular shape (in the independent case) to an elliptical shape, indicating the presence of a positive relationship between the variables. The marginal distributions remain normal, but the covariance and correlation values confirm the increased dependency between x and y. This behavior is typical for positively correlated variables in the bivariate normal distribution.

3.3 Example 3

Negatively Correlated Variables (ρ= −0.7)

In this example, we analyze the scenario where two normally distributed variables are negatively correlated with a correlation coefficient ρ=−0.7. A negative correlation means that as one variable increases, the other tends to

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decrease. In a bivariate normal distribution, negative correlation results in an elliptical scatter plot with the major axis following a downward slope, reflecting the inverse relationship.

Simulation Setup:

Mean of the first variable (μx): 0

Mean of the second variable (μy): 0

Standard deviation of the first variable (σx): 1

Standard deviation of the second variable (σy): 1

Correlation coefficient (ρ): -0.7

10,000 pairs of random variables (x and y) with a correlation of -0.7 has been generated using the bivariate normal distribution to simulate the negatively correlated variables.

Joint Scatter Plot: The scatter plot of the generated data forms an elliptical shape, but in contrast to the positively correlated case, the major axis of the ellipse is oriented downward, indicating that as one variable increases, the other decreases. The points are tightly clustered along this negative slope, showing the strength of the inverse relationship between the two variables.

Marginal Distributions: The marginal distributions of both x and y remain normal, unaffected by the negative correlation. The histograms for both variables continue to follow the standard normal distribution with the following properties:

Mean of x: Close to 0 (e.g., -0.001)

Mean of y: Close to 0 (e.g., 0.003)

Standard deviation of x: Close to 1 (e.g., 0.998)

Standard deviation of y: Close to 1 (e.g., 1.005)

Covariance and Correlation: The covariance matrix for negatively correlated variables has negative off-diagonal elements, reflecting the inverse relationship:

$$Cov(x,y) = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \quad (4)$$

The calculated covariance and correlation values are close to the input parameters:

Covariance between x and y: Close to -0.7

Correlation between x and y: Close to -0.7 (e.g., -0.699)

Elliptical Scatter Plot: The scatter plot forms an elliptical shape, with the major axis pointing in the direction of the negative correlation. The downward slope indicates an inverse relationship: as x increases, y decreases, and vice versa. The tight clustering of points along this negative axis reflects the strength of the correlation.

Marginal Distributions: The marginal distributions of x and y remain unchanged by the negative correlation. Both variables retain their normal distribution, with means close to 0 and standard deviations close to 1. The negative correlation does not affect the individual distributions but does impact their joint behavior.

Covariance and Correlation: The negative covariance indicates an inverse relationship between x and y. The correlation coefficient of -0.7 confirms that the two variables have a strong negative linear relationship. This is evident in the scatter plot where the points are tightly aligned along the negatively sloped axis.

In this example, the negative correlation between x and y is clearly reflected in the scatter plot, which shows a downward-oriented elliptical shape. The marginal distributions of the variables remain normal, and the covariance and correlation values confirm the inverse relationship. This example demonstrates how a negative correlation affects the joint distribution of two variables in a bivariate normal distribution, while keeping their individual distributions unchanged.

3.4 Example 4

Strong Positive Correlation (ρ=0.9)

In this example, we explore the effect of a strong positive correlation between two normally distributed variables with a correlation coefficient ρ=0.9. This very high positive correlation leads to points that cluster tightly along a

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positively sloped line in the scatter plot, indicating a strong linear relationship between the two variables.

Simulation Setup:

Mean of the first variable (μx): 0

Mean of the second variable (μy): 0

Standard deviation of the first variable (σx): 1

Standard deviation of the second variable (σy): 1

Correlation coefficient (ρ): 0.9

10,000 random pairs of variables (x and y) has been estimated using the bivariate normal distribution with ρ=0.9 to represent a strong positive correlation.

Joint Scatter Plot: The scatter plot of the generated data forms an elliptical shape that is tightly clustered around a positively sloped line. The high positive correlation of 0.9 means that as x increases, y also increases in a nearly linear fashion. The tight clustering of points along this positive slope indicates the strength of the linear relationship.

Marginal Distributions: The marginal distributions of both x and y remain unaffected by the strong positive correlation. Both variables follow the normal distribution, which can be observed in the histograms of the individual variables. The key statistics are as follows:

Mean of x: Close to 0 (e.g., -0.001)

Mean of y: Close to 0 (e.g., 0.003)

Standard deviation of x: Close to 1 (e.g., 0.998)

Standard deviation of y: Close to 1 (e.g., 1.002)

Covariance and Correlation: The covariance matrix for strongly positively correlated variables shows high positive off-diagonal elements, reflecting the strong linear relationship:

$$Cov(x,y) = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \quad (5)$$

The calculated covariance and correlation values are close to the input parameters:

Covariance between x and y: Close to 0.9

Correlation between x and y: Close to 0.9 (e.g., 0.899)

Elliptical Scatter Plot: The scatter plot clearly reflects the strong positive correlation. The tight clustering of points along a positively sloped line indicates that x and y increase together in a nearly linear manner. The strong linearity shows that changes in one variable can predict changes in the other with high confidence.

Marginal Distributions: Despite the strong correlation between the two variables, their marginal distributions remain unchanged. Both x and y still follow the standard normal distribution, with means close to zero and standard deviations close to one. This indicates that while the joint behavior of the two variables is strongly linear, their individual distributions remain unaffected.

Covariance and Correlation: The high positive covariance and correlation values confirm the strength of the relationship between x and y. A correlation coefficient of 0.9 indicates a very strong linear relationship between the two variables, and the covariance value reinforces this finding.

This example demonstrates how a strong positive correlation between two normally distributed variables is reflected in their joint behavior, forming a scatter plot with points tightly clustered along a positively sloped line. The marginal distributions remain unchanged despite the strong correlation, and the covariance and correlation values confirm the high degree of linearity in the relationship between the variables. This case illustrates how variables with a correlation coefficient of ρ=0.9 behave in a bivariate normal distribution.

3.5 Example 5

Conditional Distribution with ρ=0.5

Given a fixed value of one variable, the conditional distribution of the other variable is analyzed, showing how the correlation coefficient shapes the conditional normal distribution.

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4 CONCLUSION

The Bivariate Normal Distribution is a valuable statistical tool for analyzing relationships between two correlated random variables. Through five numerical examples, this study demonstrates the impact of varying the correlation coefficient on the joint behavior of the variables. Results highlight that when the correlation is zero, the variables behave independently, while positive or negative correlations lead to ellipsoidal patterns in the joint distribution. Moreover, the conditional distribution of one variable, given the other, remains normally distributed, providing a useful framework for predictive modeling. The Monte Carlo simulations used in this study further illustrate the flexibility and practicality of the Bivariate Normal Distribution in real-world applications. Whether applied to finance, engineering, or environmental data, this distribution allows for a nuanced understanding of how variables co-vary, offering insights into optimization, prediction, and decision-making processes. The results underscore the importance of the covariance matrix in determining the orientation and shape of the joint distribution, making the Bivariate Normal Distribution an indispensable tool for modeling correlated data.

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