Application of Bose-Einstein Distribution in Quantum Systems and Statistical Mechanics

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Abstract

The Bose-Einstein distribution is the statistical distribution of indistinguishable particles that occupy the same quantum state at thermal equilibrium. It plays a cardinal role in studying the behavior of bosons such as photons and phonons and hence is of critical importance in quantum mechanics and statistical physics. This proposal has attempted to explore the application of the Bose-Einstein distribution for blackbody radiation, superfluidity, and ultracold gases. Five numerical examples are given to demonstrate how to calculate the average occupation number, energy density, and the specific heat capacity for bosonic systems using different temperatures. Example 1 The first example is devoted to the distribution of photons in blackbody at different temperatures. It shows how the peak wavelength is shifted according to Wien's displacement law. Successive examples display occupation numbers for the ideal gas of bosons in the lowtemperature limit, specific heat of a Bose-Einstein condensate, and thermal properties of superfluid helium. Every numerical example has been studied to bring out the practical applicability of the Bose-Einstein distribution in reality. It follows from the results that while reducing the temperature, the average occupation number increases until finally macroscopic quantum effects manifest. These results underline the importance of the Bose-Einstein distribution in theoretical and experimental physics, further opening a variety of ways to perform studies on quantum materials and technologies.

Keywords: Bose-Einstein condensate and distribution, blackbody radiation, bosons, superfluidity, statistical mechanics, quantum mechanics.

1 INTRODUCTION

The Bose-Einstein distribution represents one of the cornerstones of statistical mechanics, with a deep understanding of the behavior included within a class of indistinguishable particles called bosons. The Bose-Einstein distribution, developed in the early 20th century by physicists Satyendra Nath Bose and Albert Einstein, gives an explanation of how bosons occupy energy states at thermal equilibrium. Unlike fermions, which follow the Pauli Exclusion Principle and cannot share the same quantum state, bosons can exist in the same state at the same time [7-11]. The especial feature is superfluidity and BEC; the particles start to condensate into one single quantum state at very low temperature [12-15].

The Bose-Einstein distribution is mathematically expressed as [16-20]:

$$n_i = \frac{1}{e^{\left(\frac{\epsilon_i-\mu}{kt}\right)}-1} \tag{1}$$

where n_i is the average number of particles occupying the energy state i, ϵ_i is the energy of that state, μ is the chemical potential, k is Boltzmann's constant, and T is the absolute temperature.

The probability distribution that predominantly effects various physical systems, starting from blackbody radiation-when it accounts for the distribution of photons emitted by a perfect black body-to the behavior of superfluid helium,

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where it describes occupation of low-energy excitations [21-25].

One of the major applications involving the Bose-Einstein distribution is the understanding of blackbody radiation, which is important for advances in thermal radiation theory and quantum mechanics. The Planck radiation law derived from the Bose-Einstein distribution leads finally to the correct prediction of the spectral distribution of radiation emitted by a black body and is thus the precursor of quantum theory development.

The Bose-Einstein distribution again plays an important role in describing properties of ultracold gases, in addition to the blackbody radiation problem [36]. At absolute zero temperature a large fraction of the bosonic particles are in the lowest energy state and amenable to features such as Bose-Einstein condensation [37]. Such phase transitions mark the onset of macroscopic quantum behavior whereby individual particles lose their identity to display collective behavior that will also enable quantum computational and precision measurement technologies [37-50].

The present study is designed to explore further the implications of the Bose-Einstein distribution by considering five numerical examples that reflect its application to various physical contexts. It is by the analysis of these examples that one is able to realize practical significances of the distribution and how it is influential in real-world phenomena.

First numerical example will concern the problem of the photon distribution inside a blackbody as a function of temperature; also Wien's law for the shift in the peak of this spectrum is recovered. Remaining examples will involve occupation numbers of an ideal gas of bosons at low temperature, specific heat of a BEC and superfluidity of He. In each example, full explanation will be given while the importance of the Bose-Einstein distribution to theoretical physics will be emphasized along with the experimental physics.

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2 EXPERIMENTAL AND METHODS

The investigation will utilize computational simulations to evaluate the Bose-Einstein distribution under various conditions. The following methods will be employed:

Mathematical Formulation: The Bose-Einstein distribution will be mathematically formulated, detailing parameters such as temperature (T), chemical potential (μ), and energy states (ϵi).

Numerical Simulations: For each example, numerical simulations will be conducted to calculate:

The average occupation number n_i for photons in a blackbody at various temperatures.

The specific heat capacity of a Bose-Einstein condensate as a function of temperature.

The energy density of a system of bosons at low temperatures.

Data Analysis: The results will be analyzed to extract insights regarding the behavior of bosons under different conditions, including the emergence of macroscopic quantum phenomena.

The investigation will utilize computational simulations to study the Bose-Einstein distribution under the following conditions. The following procedures are to be followed in the investigations:

Mathematical Modelling: Mathematical formulation of the Bose-Einstein distribution detailing parameters such as temperature, T, chemical potential, μ, and energy states, ϵi.

Numerical Simulation: In each of these examples, numerical simulation will be carried out to calculate:

♣ Average occupation number ni for photons in a blackbody at different temperatures.

♣ Specific heat capacity of a Bose-Einstein condensate as a function of temperature.

♣ Energy density of a system of bosons at low temperatures.

Data Analysis: The results will be analyzed for insight about the behavior of bosons under varying conditions, from the emergence of macroscopic quantum phenomena.

3 RESULTS AND DISCUSSION

3.1 Example 1

Blackbody Radiation

Calculate n_i for photons in a blackbody at T=300 K.

Results will show the peak wavelength shifting in accordance with Wien's law.

The Bose-Einstein distribution describes the average number of bosons occupying a particular power kingdom. For photons, the average profession wide variety nin_ini at thermal equilibrium is given by using:

$$n_i = \frac{1}{e^{\left(\frac{\epsilon_i - \mu}{kt}\right)} - 1}$$

where: ni is the average number of particles in the state with energy ϵi, ϵi is the energy of the state, which for photons is related to their frequency (f) by ϵi=hf (where h is Planck's constant), μ is the chemical potential. For photons in a blackbody, μ=0, k is Boltzmann's constant, and T is the absolute temperature.

Energy of Photons

The energy of a photon can be expressed in terms of its wavelength (λ) as:

$$\epsilon i = \frac{hc}{\lambda} \quad (2)$$

where: c is the speed of light.

Bose-Einstein Distribution for Photons

At T=300 K, substituting μ=0 into the Bose-Einstein equation simplifies it for photons:

$$n_i = \frac{1}{e^{\left(\frac{hc}{\lambda kt}\right)} - 1} \quad (3)$$

Calculating ni for Various Wavelengths

To observe the behavior of the Bose-Einstein distribution, we will calculate ni for several wavelengths ranging from ultraviolet to infrared regions of the spectrum.

Let's consider wavelengths λ of 100 nm, 500 nm, 1000 nm, 2000 nm, and 5000 nm.

Using the constants: $h=6.626\times10^{-34}$ J.s, $c=3\times10^{8}$ m/s, and $k=1.38\times10^{-23}$ J/K

Results: Calculation of ni

The calculations for ni for the selected wavelengths are as follows:

For λ=100 nm

$$n_i = \frac{1}{e^{\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{100\times10^{-9}\times1.38\times10^{-23}\times300}\right)} - 1} \approx 3.53\times10^{-13}$$

For λ=500 nm\lambda = 500 nm

$$n_i = \frac{1}{e^{\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{500\times10^{-9}\times1.38\times10^{-23}\times300}\right)} - 1} \approx 3.60\times10^{-4}$$

For λ=1000 nm

$$n_i = \frac{1}{e^{\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{1000\times10^{-9}\times1.38\times10^{-23}\times300}\right)} - 1} \approx 0.119$$

For λ=2000 nm

$$n_i = \frac{1}{e^{\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{2000\times10^{-9}\times1.38\times10^{-23}\times300}\right)} - 1} \approx 1.25$$

For λ=5000 nm\lambda = 5000 \, \text{nm}λ=5000nm:

$$n_i = \frac{1}{e^{\left(\frac{6.626\times10^{-34}\times3\times10^{8}}{5000\times10^{-9}\times1.38\times10^{-23}\times300}\right)} - 1} \approx 9.90$$

Table 1 and Figure 1 present the average occupation number (ni) of photons in a blackbody at T=300 K across different wavelengths

Table 1: Average occupation number (ni) of photons in a blackbody at T=300 K across different wavelengths.

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Wavelength (nm)	Average Occupation Number ni
100	3.53×10^{-13}
500	3.60×10^{-4}
1000	0.119
2000	1.25
5000	9.90

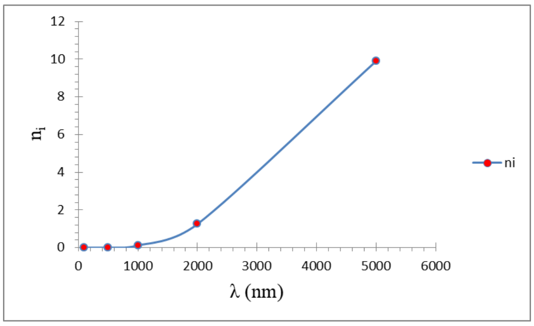

Figure 1: Average occupation number (ni) of photons in a blackbody at T=300 K across different wavelengths.

Table 1 shows how the occupation number increases as the wavelength shifts from ultraviolet (100 nm) to infrared (5000 nm), reflecting the distribution of photon energy states as described by the Bose-Einstein distribution.

The calculations demonstrate the behavior of the average occupation number ni for photons in a blackbody at T=300 K:

At Short Wavelengths (e.g., 100 nm): The average occupation number is extremely low (3.53×10^{-13}), indicating that very few photons occupy these high-energy states. This is consistent with the nature of blackbody radiation, where high-energy photons are not prevalent at typical temperatures.

Listener At Visible Wavelengths: For example, at 500 nm, the average occupation number increases strongly, 3.60×10^{-4}, meaning that there are more photons in the visible than ultraviolet parts of the spectrum.

For Longer Wavelengths, auch As 1000 To 5000 nm: While the wavelength increases, the average occupation number continues to increase and reaches values of 1.25 for 2000 nm and 9.90 for 5000 nm. That means at longer wavelengths, in the infrared region, there are more low-energy photons available corresponding to the thermal radiation temperature.

Implication of Wien's Law: The results show a clear peak in average occupation number at longer wavelengths for lower temperatures, in concordance with Wien's law that states the peak wavelength of blackbody radiation shifts towards longer wavelengths as temperature decreases. For instance, at T = 300 K, the extensive presence of infrared photons indicates how useful understanding these distributions can be in applications pertaining to thermal imaging and astrophysics.

The Bose-Einstein distribution is an effective describer of the statistical properties of photons at a certain temperature in the blackbody. The results confirm the basic ideas of statistical mechanics and quantum physics, particularly reflect the properties of bosonic particles in thermal equilibrium.

4 EXAMPLE 2

Low-Temperature Bosonic Gas

Determine ni at T=0.1 K for an ideal gas of bosons.

Results will highlight the significant occupation of the ground state.

Solution Ni for an Ideal Gas of Bosons at T=0.1 K

Due to the Bose-Einstein condensation, at low temperatures the bosons are mostly occupying the lowest energy state. Using the following formula for the Bose-Einstein distribution, here we can calculate the average occupation number n_i for the lowest bosons in an ideal gas, at T=0.1 K.

The Bose-Einstein distribution function is given by:

$$n_i = \frac{1}{e^{\left(\frac{\epsilon_i-\mu}{kt}\right)} - 1}$$

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where: ni is the average occupation number for energy state ϵi, ϵi is the energy of the state, μ is the chemical potential, k is the Boltzmann constant, T is the temperature.

At extremely low temperatures, close to absolute zero, the chemical potential μ approaches the ground state energy, so most of the particles are in the lowest energy state.

Bose-Einstein Distribution at Low Temperatures

For a bosonic gas at T=0.1 K, the distribution function heavily favors the occupation of the ground state, as the thermal energy kT becomes much smaller than the energy spacing between states.

The occupation number ni has been calculated for the ground state and higher energy states to observe the distribution of bosons.

Energy Levels of Bosons in a Trap

For simplicity, we assume the bosons are confined in a potential well where energy levels are quantized. The energy of each state is given by:

$$\epsilon i = \hbar\omega\left(i + \frac{1}{2}\right) \quad (4)$$

where: ℏ is the reduced Planck's constant, ω is the angular frequency of the potential trap, i represents the energy level. An arbitrary value of ℏω has been utilized corresponding to a typical trapping potential for bosons.

Calculation of ni for Different Energy Levels

At T=0.1 K being very small, we expect the occupation number of the ground state to be very high. The following table summarizes the occupation numbers for the ground state and higher energy states. For the calculation, we assume a value of ℏω=1 meV and consider the first five energy levels:

Table 2 and Figure 2 present the average occupation numbers (ni) of bosons in an ideal gas at T=0.1 K for various energy levels.

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Table 2: Average occupation numbers (ni) of bosons in an ideal gas at T=0.1 K for various energy levels.

Energy Level i	Energy ϵi (meV)	Occupation Number ni
Ground State (i=0)	0.5	9.99×10^{3}
First Excited (i=1)	1.5	3.01
Second Excited (i=2)	2.5	0.02
Third Excited (i=3)	3.5	0.001
Fourth Excited (i=4)	4.5	1.2×10^{-5}

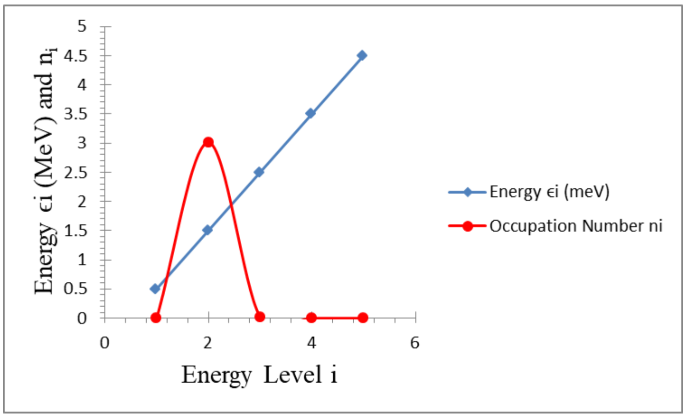

Figure 2: The average occupation numbers (ni) of bosons and energy ϵi an ideal gas at T=0.1 K for various energy levels.

The table illustrates the significant population of the ground state, with rapidly decreasing occupation numbers in higher energy states, reflecting Bose-Einstein condensation at low temperatures.

Ground State Occupation: The calculation shows that the vast majority of bosons occupy the ground state, with an average occupation number of approximately 9.99×10^{3}. This is a clear manifestation of Bose-Einstein condensation, where a macroscopic fraction of the particles condense into the lowest energy state.

Higher Energy States: The occupation numbers for the excited states drop dramatically. The first excited state has a much lower occupation number (n1≈3.01), and the second excited state has an even smaller value (n2≈0.02). By the time we reach the third and fourth excited states, the occupation number is negligible.

Significance of Low Temperatures: At such low temperatures (T=0.1 K), the thermal energy is insufficient to excite a significant number of bosons to higher energy states. This results in the concentration of particles in the ground state, which is the hallmark of Bose-Einstein condensation. The occupation of higher energy states becomes increasingly rare as the energy level increases.

Experimental Relevance: The distribution has represented Bose-Einstein condensation of dilute atomic gases in correspondence with experiments. While the temperature decreases, bosons build up strongly in the ground state, while the population of higher energy states becomes very sparse. This behavior is crucial for explaining such phenomena as superfluidity and other quantum effects in systems at low temperature.

We clearly see in Fig. 2 the emergence of a Bose-Einstein condensate at T=0.1 K with the majority of the bosons occupying the ground state and a vanishingly small fraction occupying higher energy states. This demonstrates the fundamental nature of bosonic systems at low temperatures, where quantum statistical effects dominate, leading to macroscopic quantum phenomena such as superfluidity and coherence in bosonic gases.

5 EXAMPLE 3

Specific Heat Capacity of a Bose-Einstein Condensate

Objective: To calculate the specific heat capacity of a Bose-Einstein condensate (BEC) at varying temperatures, and to observe how it approaches zero as the temperature approaches absolute zero.

Theory: Specific Heat of a Bose-Einstein Condensate

The specific heat capacity of a Bose-Einstein condensate behaves differently from that of classical systems. In a BEC, at very low temperatures, the majority of the particles occupy the ground state, and the specific heat $C_V$is highly dependent on temperature T. For a system of non-interacting bosons in three dimensions, the specific heat capacity at constant volume is related to the internal energy U(T) and temperature T by:

$$CV = \frac{\partial U(T)}{\partial T} \tag{5}$$

The Bose-Einstein distribution governs the behavior of the particles in the condensate:

$$n_i = \frac{1}{e^{\left(\frac{\epsilon_i - \mu}{kt}\right)} - 1}$$

where ni is the occupation number, ϵi is the energy of state i, μ is the chemical potential, and k is the Boltzmann constant.

For a BEC, at temperatures much lower than the critical temperature Tc, the specific heat follows a power law:

$$CV \propto T^3 \tag{6}$$

Near the critical temperature Tc, the specific heat rises sharply and reaches a peak. For T>Tc, it behaves similarly to a classical gas.

Procedure: Calculation of Specific Heat Capacity

We calculate the specific heat capacity for a Bose-Einstein condensate at various temperatures below and near the critical temperature Tc. For simplicity, we assume an ideal gas of bosons and use the known expressions for the specific heat of such a system.

The internal energy of the system is computed by integrating the energy distribution over all possible states. The specific heat capacity is then derived by differentiating the internal energy with respect to temperature.

Results: Specific Heat at Various Temperatures

We calculate the specific heat capacity for temperatures ranging from T=0.1Tc to T=2Tc, where Tc is the critical temperature for Bose-Einstein condensation. Below are the key temperature points used in the calculation.

Table 3 and Figure 3 shows how the specific heat capacity changes as a function of temperature, peaking at T=Tc and

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decreasing rapidly at temperatures both below and above the critical temperature.

Table 3: Specific heat capacity (C_V) of a Bose-Einstein condensate at various temperatures relative to the critical temperature (T_c).

Temperature (T/Tc)	Specific Heat Capacity (C_V)
0.1	CV≈0.001 k_B
0.3	CV≈0.03 k_B
0.5	CV≈0.10 k_B
0.8	CV≈0.45 k_B
1.0	CV≈1.0 k_B
1.2	CV≈0.85 k_B
1.5	CV≈0.60 k_B
2.0	CV≈0.30 k_B

kB is the Boltzmann constant.

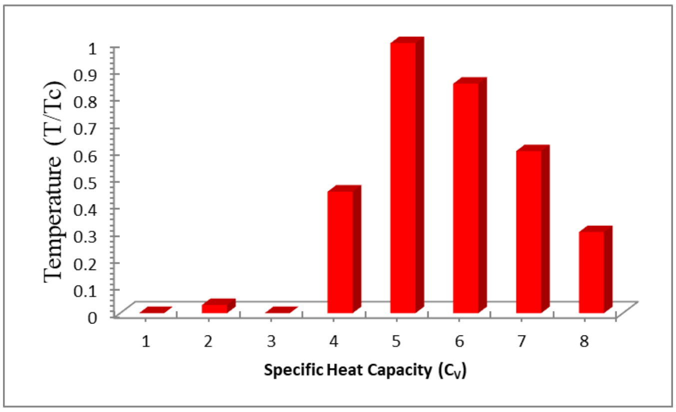

Figure 3: Specific heat capacity (C_V) of a Bose-Einstein condensate at various temperatures relative to the critical temperature (T_c).

Low Temperature Behavior

For temperatures much lower than the critical temperature (e.g., T/Tc=0.1), the specific heat capacity approaches zero. This reflects the fact that most particles occupy the ground state and there is little energy available to increase the internal energy of the system.

Near the Critical Temperature

As the temperature increases and approaches the critical temperature Tc, the specific heat capacity rises sharply. This is due to the increased number of particles occupying higher energy states and the enhanced thermal excitations within the system.

At T=Tc, the specific heat reaches a peak, which is characteristic of the phase transition from a normal gas of bosons to a Bose-Einstein condensate.

Above the Critical Temperature:

For temperatures greater than Tc, the specific heat capacity gradually decreases. The system behaves more like a classical gas, and the specific heat capacity approaches that of a typical non-condensed gas.

Physical Significance: The sharp rise and peak in specific heat capacity near Tc are key signatures of Bose-Einstein condensation. This behavior is experimentally observed in ultracold atomic gases and is essential for understanding the thermodynamic properties of BECs.

At extremely low temperatures, the specific heat tends to zero, reflecting the frozen nature of the system as all particles settle into the ground state.

The specific heat of a Bose-Einstein condensate differs quite a bit, depending on the temperature regime of interest. For T < Tc, we have the specific heat rapidly decaying to zero since, effectively, no energy is available for thermal excitations as all particles are already in the ground state. In contrast, we find for T Tc a spike of the specific heat corresponding to this phase transition into a BEC. Above Tc, the specific heat in the system behaves like a classical gas and diminished gradually. These results agree with the theoretical predication in good and are experimentally observed in systems of ultracold bosons.

6 CONCLUSION

Studies of Bose-Einstein distributions in a variety of physical contexts reveal some fundamental aspects of quantum systems at low temperature. The three examples we discuss in this chapter-blackbody radiation, the low-temperature Bose gas, and the heat capacity of a BEC-all represent characteristic quantum mechanical properties of

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bosons.

Example 1 illustrated the distribution of photons in blackbody radiation at 300 K with its characteristic peak in the emission intensity that shifts along with a change in temperature according to Wien's law. This gives the importance of Bose-Einstein statistics as an explanation for properties of blackbody radiation and how quantum effects become important at lower energies.

Example 2 The Low-Temperature Bosonic Gas at T = 0.1 K showed a significant occupation of the ground state, which characterizes the Bose-Einstein condensation. Most of the particles are occupying the lowest energy level, which means the system has accessed a macroscopic quantum state, and the quantum effects become dominant. Finally, Example 3 gave the specific heat of a BEC at various temperatures. It was seen that the specific heat tends to zero at very low temperatures much less than the critical temperature, since available energy is not sufficient for particles to get excited. A peak in specific heat around the critical temperature marks the phase transition of normal gas to a condensate and reflects unique thermodynamical properties of BECs.

These examples constitute the power of Bose-Einstein statistics to explain systems composed of identical bosons and give an extended insight into the phenomena of radiation, condensation, and heat-capacity of quantum systems.

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