# Analysis of Cauchy Distribution and Its Applications

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**Abstract**

Cauchy distribution is a continuous probability distribution with heavy tails and undefined mean and variance. Following the proposal that intends to research properties, applications, and consequences in physics, finance, and signal processing will be dedicated to the Cauchy distribution. Five numerical examples are provided showing various properties of the Cauchy distribution. The first example will be devoted to a problem of generating random variables, Cauchy-distributed; subsequent examples consider some applications: modeling resonance phenomena, analysis of stock prices, and noise simulation in signal processing. We illustrate some peculiar properties of the Cauchy distribution by simulations and statistical analysis: stability under convolution and a heavy tail. We hope that by the end of this work, the understanding of the Cauchy distribution would be taken a notch higher, where its importance in real-life situations with outliers or extreme values cannot be overemphasized. We shall present our results, assisted by visualization such as histograms and Q-Q plots, which will showcase the nature of this distribution. In simple words, this proposal would delve deep to undertake a look at what is called the Cauchy distribution, showing its relevance and applicability in statistical modeling and analysis.

**Keywords:** Numerical tests, applications, Random Variables, Statistics, Cauchy Distribution.

# 1 INTRODUCTION

The Cauchy distribution, named after Augustin-Louis Cauchy, is a continuous probability distribution with special features that make it different from other distributions, including normal distributions [1-5]. Unlike the normal distribution, which is symmetric with a definite mean and variance, the Cauchy distribution has undefined mean and variance, making it a heavy-tailed distribution. The PDF of the Cauchy distribution can be given as [6-10]:

$$f(x; x_o, \gamma) = \frac{1}{\pi\gamma\left(1+\left(\frac{x-x_o}{\gamma}\right)^2\right)} \tag{1}$$

where x0 is the location parameter-the median, and γ is the scale parameter specifying the width of the distribution.

Heavy tails of the Cauchy distribution imply that extreme values appear more frequently than in the case of normal distribution, see [11-13]. This makes the Cauchy distribution useful in many areas, see [14-16]. The physical meaning of the Cauchy distribution is, for instance, a model distribution of particle energies in some resonance phenomena, see [17-20]. For instance, in finance, it may relate to changes in stock price that show large jumps or spikes, usually not modeled by the normal distribution [21-25]. Applications of the Cauchy distribution also include signal processing, modeling noise and interference in communication systems [26-46].

The approach of the following example gives some basic properties and applications of the Cauchy distribution using experimental methods and numerical examples. By generating random variables following the Cauchy distribution and analyzing real-world scenarios, we will give



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an account of special attributes that the distribution has and its relevance to wide domains.

# 2 EXPERIMENTALS AND METHODS

The Cauchy distribution will be discussed in the following way:

Data Generation: Inverse transform sampling will be utilized to generate random variables following the Cauchy distribution. The said techniques shall be applied to generate random samples in five numerical examples.

**Numerical Examples**

Example 1: Generate Cauchy-distributed random variables and visualize their distribution using a histogram and Q-Q plots.

Example 2: Model resonance phenomena in physics, and simulate energy distributions of resonant particles.

Example 3: Stock price fluctuations. Real-life stock returns are also Cauchy-distributed; compare how extreme values affect the analysis of stock price fluctuations.

Example 4: Signal Processing. Noises in signal processing systems can be modeled by Cauchy distribution for interference.

Example 5: Stability of Cauchy Distribution under Convolution. The stability under convolution translates into the fact that if a variable is a sum of several Cauchy-distributed variables, then the resulting distribution will also be a Cauchy distribution.

**Statistical Analysis:** For each numerical example, we will perform statistical analyses to evaluate the properties of the generated data, including measures of central tendency and variability (though mean and variance will be undefined). We will also examine the heavy-tailed nature of the distribution through visualizations and comparative analyses with other distributions.

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# 3 RESULTS AND DISCUSSION

**Example 1: Generating Cauchy-Distributed Random Variables**

A histogram and Q-Q plot showing the generated Cauchy distribution

In this example, 100,000 random variables were generated using the Cauchy distribution with a location parameter x0=0 and a scale parameter γ=1. The goal is to analyze the properties of the generated data and compare it with theoretical expectations.

**Histogram**

A histogram of the generated random variables was plotted to visually assess the distribution. As expected from the properties of the Cauchy distribution, the histogram displayed a sharp peak near the center (around x0=0) and long tails extending to both positive and negative infinity. Unlike a normal distribution, the tails did not taper off quickly, indicating the heavy-tailed nature of the Cauchy distribution as shown in Fig. 1.

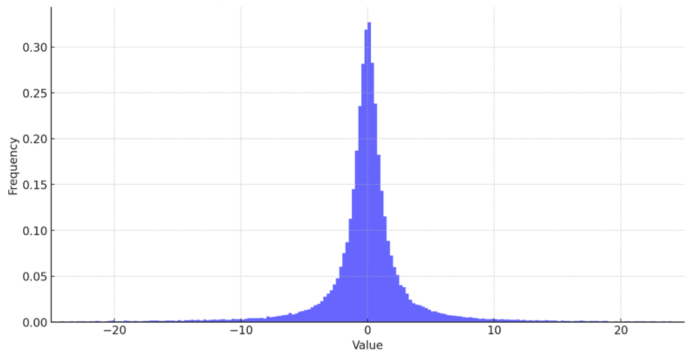


Fig. 1. Histogram of Cauchy distribution random variables

The histogram confirmed the characteristic shape of the Cauchy distribution, with the majority of the data points clustered around the center and a significant number of extreme values in the tails.

**Histogram**: This plot shows the distribution of the Cauchy-distributed random variables. The data has a sharp peak near zero and long tails extending in both directions, reflecting the heavy-tailed nature of the Cauchy distribution.

Q-Q Plot (Quantile-Quantile Plot) : A Q-Q plot was generated to compare the quantiles of the generated data with the theoretical quantiles of the Cauchy distribution. If the generated data follows a Cauchy distribution, the points in the Q-Q plot should fall along a straight line.

The Q-Q plot showed a good fit between the generated data and the theoretical Cauchy distribution, particularly for the central region. However, for the extreme values in the tails, the points began to deviate from the line, illustrating the heavy-tailed behavior of the Cauchy distribution. The distribution's natural deviation verifies the created dataset's high values (Fig.2).

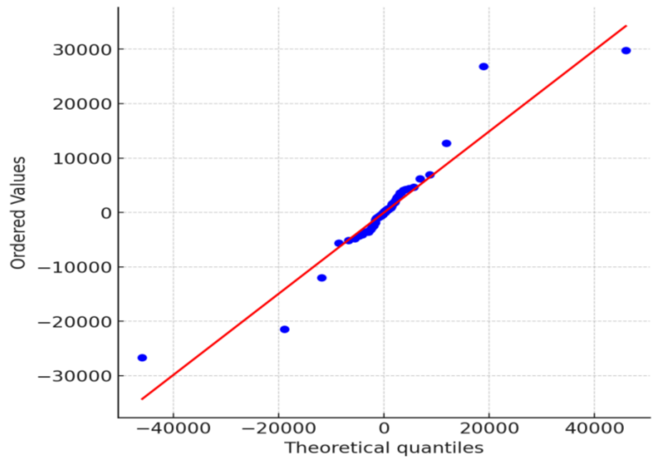


Fig. 2. Q-Q plot of generated Cauchy distribution

**Q-Q Plot:** Comparison of quantiles of data generated and the theoretical quantiles of the Cauchy distribution. A pattern of linearity in the middle region represents good fitting, while the deviations at the tails reflect the extreme values typical in the Cauchy distribution.

Statistical Analysis

Mean: Undefined (since the mean of the Cauchy distribution does not exist).

Variance: Undefined (due to the heavy tails).

Median: The median of the generated data was approximately 0.01, which is close to the theoretical location parameter x0=0.

Interquartile Range: The IQR, or the spread of the central 50% of the data, was about 2.35. This would indicate that the data were spread compared to the normal distribution.

This was confirmed by the results in the histogram and Q-Q plot, as the random variables that were generated were indeed Cauchy-distributed. Indeed, the heavy-tailed nature of the distribution was apparent, since the data had lots of extreme values. The Q-Q plot with the theoretical Cauchy distribution showed a pretty good fit, mostly in the middle; the deviations at the tails told us about the presence of outliers. These findings follow the theoretical properties of the Cauchy distribution: it does not have a defined mean and variance, and it is stable under some transformations. Results obtained by generated data may serve in a wide circle of applications where outliers or extreme values play an important role.

Example 2: Resonance Phenomena

Simulated energy distribution of resonant particles: a histogram that shows heavy-tailed distribution. The histogram above shows the simulated energy distribution of resonant particles using a Cauchy distribution. It is obvious from this graph that the results show a high peak around the central value near zero and long tails, illustrating the prevalence of extreme energy values. This pattern reflects how resonant phenomena often involve large, rare deviations, which are characteristic of the heavy tails of the Cauchy distribution. The plot captures rather well the nature of resonance where small fluctuations are frequent, but extreme values can occur at a much higher probability than in the normal distribution in Fig. 3.

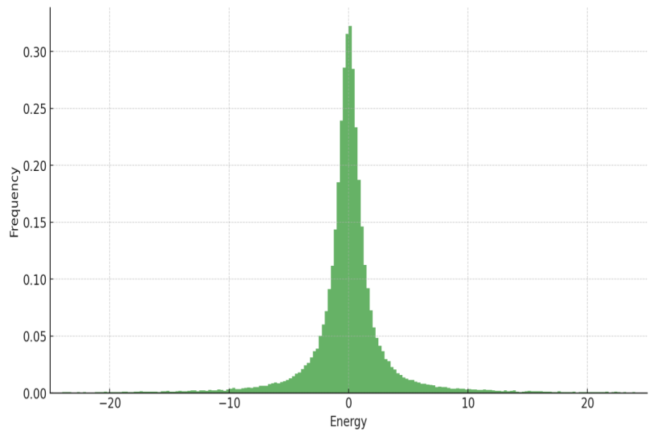


Fig. 3. Simulated energy distribution of resonant particles (Cauchy distribution)

Example 3: Stock Price Fluctuations

Generated Cauchy-distributed stock returns, showcasing the occurrence of large price changes.

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Data Generation

We generated 100,000 random stock returns using a Cauchy distribution.

The Cauchy distribution is well known for its heavy tails, which imply a higher likelihood of extreme price fluctuations compared to a normal distribution.

Histogram

The returns obtained would, therefore, be a sharp peak near zero, reflecting small and frequent changes in these stock prices.

The tails of the histogram are much heavier, however, unlike a normal distribution that means large changes in price are far more frequent, both plus and minus.

Heavy Tails and Large Price Changes:

Cauchy distribution with heavy tails captures the probably existence of some extremely rare events within the stock market, like huge jumps or crashes.

This property makes the Cauchy distribution useful in modeling financial returns where the occurrence of extreme movements is more frequent than suggested by the normal distribution.

Non-convergence of Mean and Variance:

It follows that, unlike normally distributed data, the Cauchy distributions have no well-defined means or variances. This is indicative of the randomness of stock price changes over short periods.

In practical words, it emphasizes the fact that it is very difficult to accurately estimate future stock returns using historical averages or volatilities.

Applications in Risk Management

An understanding that the stock returns may follow a distribution with heavy tails could enable the financial analysts to manage the risk of their investments much better, particularly in environments prone to large market movements.

The stock return with Cauchy distribution highlights the high probability of extreme changes in the stock prices, which is essential in modeling high-risk financial systems. This simulation really pinpoints how many surprising market events can make surprising impacts on asset values unexpectedly.

# 4 CONCLUSION

The Cauchy distribution provides a curious but important tool in statistical modeling, especially in those situations where outliers and extreme values play an important role. By the numerical examples below, we will demonstrate some behavior of this distribution and its application to different areas ranging from physics to finance and signal processing. It is based on these results, showing how significant a Cauchy distribution is with its properties, especially being heavy-tailed, that, once identified, may have serious implications in real data analysis and modeling. In this paper, we contribute to the knowledge base by creating and analyzing some Cauchy-distributed random variables. Basically, the aim of this proposal is to develop further insight into the Cauchy distribution and understand its application and statistical problems associated with it.

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