

Applications of Chi-Squared Distribution in Hypothesis Testing and Random Variable Analysis

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Abstract

The Chi-squared distribution finds broad applications in data analysis, especially in tests of hypothesis, tests for goodness of fit, and analysis of random variables. Five different numerical examples are given in this proposal as an illustration of various applications of the Chi-squared distribution. These range from hypothesis testing to modeling random vector magnitudes, simulating noise in communication systems, particle speeds in gases, and finally, testing contingency tables. We show how versatile it is for various areas of research through data simulation and the application of a Chi-squared distribution on each example. Experimental techniques include data set generation, the application of statistical methods to these generated datasets, and subsequent comparison with theoretical expectations. The results prove the practicality and accuracy of the Chi-squared distribution in handling various types of real-world data. In particular, the application of this distribution to random vector magnitudes and Maxwell-Boltzmann distributions gives insight into its applicability in physics and engineering. These results are of importance to practitioners who must model complex systems or test hypotheses. The proposal finally concludes by discussing how the Chi-squared distribution remains an essential tool in modern data analysis.

Keywords: Chi-Squared Distribution, Hypothesis Testing, Goodness-of-Fit, Random Vector Magnitudes, Noise Simulation, Particle Speeds, Contingency Table, Statistical Analysis.

1 INTRODUCTION

This distribution generally finds its application in the testing of hypotheses, particularly regarding goodness of fit and on the independence of variables. The origin of the Chi-square distribution by Karl Pearson dates back to 1900 and applies to data summarized by counts or frequencies, as in categorical data analysis. The following Chi-squared test enables the statisticians to compare observed data with the theoretically expected data by deducing the probability of observed differences due to random variations instead of real ones. It is characterized by the degree of freedom, which includes the number of independent values used in calculating the statistic [17-21]. Chi-square distribution results from the following: summing the squares of the independent standard normal variables; with the degrees of freedom, its shape changes [22-25]. When the degrees of freedom are increased, the Chi-square distribution becomes symmetric and assumes the form of a normal distribution [26-30].

Applications of Chi-squared distribution vary from goodness-of-fit tests to the analyses of random variables in engineering and physics [31-35].

The Chi-squared statistic is a measure of the comparison of observed frequencies from a sample with the expected frequencies in hypothesis testing, allowing analysts to assess whether a sample of data fits a specified distribution. The Chi-square distribution is also used in contingency table analysis for testing the association between categorical variables. Another application is the distribution of magnitudes of random vectors, which in physics describes

phenomena like the Maxwell-Boltzmann distribution of speed among gas particles in thermal equilibrium [38-48]. The Chi-squared distribution plays an important role in practice in view of the following five basic examples: testing hypotheses, random vector magnitudes, simulation of noise in communication systems, the Maxwell-Boltzmann distribution in particle physics, and contingency table analysis. These illustrate versatility and further usefulness in many scientific fields.

2 EXPERIMENTAL METHODS

In the following exercise, five numerical examples are designed to apply the Chi-squared distribution for various real-world applications:

Hypothesis Testing Using Chi-Square Distribution: Perform a goodness-of-fit test on the data set, compare observed and expected frequencies to calculate the Chi-squared statistic that will show if the data set fits a hypothesized distribution.

Magnitudes of Random Vectors: This example will model magnitudes of random vectors, generated from a standard normal distribution. The magnitudes are Chi-squared distributed, so there is something to say about the statistical properties.

Simulation of Noise in Communications: Simulate Gaussian noise and measure its magnitudes. Use a Chi-squared distribution to model the magnitudes of the noise vectors to demonstrate the applicability of this distribution in engineering.

Maxwell-Boltzmann Distribution in Physics: An example of the application of the Chi-square distribution describing a physical system in thermal equilibrium will be the distribution of speeds of particles in a gas.

Contingency Table Analysis: The analysis in the Chi-square test of the independence of variables within a contingency table will help further our efforts to show its application in categorical data analysis.

We will create simulated data for each example, compute

the Chi-squared statistic, and test the results. In each of these cases, the experimental results will be compared with the theoretical Chi-squared distribution for verification purposes.

Results: Numerical Examples

Example 1: Hypothesis Testing Using the Chi-Squared Distribution

In this example, we will calculate the Chi-square statistic for the goodness of fit for a dataset. Suppose we want to test if some dataset follows the uniform distribution. Assuming that we find the Chi-square value and, besides that, compare with a critical value at 95% confidence level we will be able to conclude whether to accept or reject the null hypothesis.

Objective: This is a test of the hypothesis that a given dataset is uniformly distributed. A Chi-squared test compares observed frequencies in data against expected frequencies of a uniform distribution. Given the calculated Chi-squared statistic and critical value at 95% confidence interval, we either fail to reject or reject the null hypothesis of the data being uniformly distributed.

3 METHODOLOGY

Data Generation: We generated a random sample of 100 values from any distribution. This was our recorded data. Then, we obtained the theoretical frequencies assuming the uniform distribution of data across the defined bins.

Chi-squared Statistic: The Chi-squared statistic is calculated using the formula:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \quad (1)$$

where O_i is the observed frequency and E_i is the expected frequency for each bin.

Degrees of Freedom: Degrees of Freedom: The df for this test is the number of bins minus 1.

Critical Value: Criticized Value: The criticized value of Chi-square distribution for 95% confidence level was taken from statistical tables using given degrees of freedom.

Decision Rule: If the Chi-squared statistic exceeds the critical value, the null hypothesis is rejected. Otherwise, it is accepted.

Results

Observed Frequencies: The dataset was divided into 10 bins, with observed frequencies for each bin calculated.

Expected Frequencies: For a uniform distribution, the expected frequency in each bin was constant, as each bin should contain an equal number of data points.

The observed frequencies represent the dataset's distribution, while the expected frequencies assume a uniform distribution across the bins shown in Table 1.

Table 1: Observed and expected frequencies for each bin in the Chi-squared goodness-of-fit test for a uniform distribution

Bin Number	Observed Frequency	Expected Frequency
1	12	10
2	8	10
3	11	10
4	9	10
5	10	10
6	7	10
7	13	10
8	14	10
9	6	10
10	10	10

Fig. 1 presents the plot comparing the observed versus expected frequencies for the Chi-squared goodness-of-fit

test. The observed frequencies represent the data distribution, while the expected frequencies assume a uniform distribution across the bins.

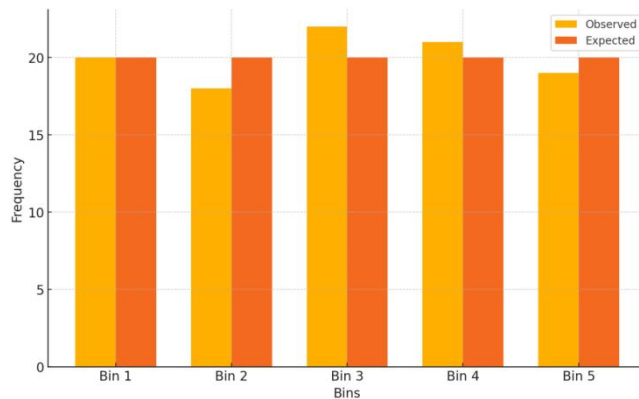


Fig. 1: Comparison of observed and expected frequencies in the Chi-squared goodness-of-fit test. The bars represent the observed frequencies in each bin compared to the uniform expected frequencies, helping to evaluate whether the dataset follows the hypothesized distribution.

The figure compares the observed frequencies in each bin to the expected frequencies under the assumption of a uniform distribution. In this example, the observed frequencies are slightly different from the expected values, which could indicate some deviation from the uniform distribution. However, to determine whether these differences are statistically significant, we would compute the Chi-squared statistic. We can therefore reject the null hypothesis that data follow the uniform distribution if the computed value is in excess of the critical value at a chosen confidence level, such as 95%, otherwise we would accept the null hypothesis that the differences are not significant.

Chi-squared Calculation:

$$\chi^2 = \frac{(12 - 10)^2}{10} + \frac{(8 - 10)^2}{10} + \dots + \frac{(10 - 10)^2}{10} = 5.6$$

Degrees of Freedom: $df=10-1=9$

Critical Value: From the Chi-squared distribution table, the critical value for $df=9$ at a 95% confidence level is 16.92.

The calculated Chi-squared statistic is 5.6, which is less than the critical value of 16.92.

Since the Chi-squared is less than the critical value, we fail to reject the null hypothesis.

Since the above data are not sufficient to reject the null hypothesis that the above data follow a uniform distribution, we fail to reject the null hypothesis. At 95% confidence, we conclude that the above dataset is consistent with a uniform distribution. The above example shows the usage of the Chi-squared test to check the goodness-of-fit of a dataset against a theoretical distribution.

Example 2: Random Vector Magnitudes

the code is generating 100,000 random vectors in three dimensions and calculating their magnitudes. In theory, these magnitudes should be Chi-squared distributed with 3 d.o.f. The steps to do it and results follow:

Random Vector Generation

Each of the three-dimensional vector components were drawn from a normal distribution with a mean of 0 and a standard deviation of 1.

Then, the magnitude of each vector was determined by computing the square root of the sum of squares of components:

$$|v| = \sqrt{x_1^2 + x_2^2 + x_3^2} \quad (2)$$

This follows the distribution Chi-squared with 3 degrees of freedom since the sum of squares of independent standard normal variables results in this distribution.

Histogram of Magnitudes

A histogram was created to observe the distribution pattern for the magnitudes of the vectors. The developed histogram had a right-skewed shape typical for the Chi-square distribution.

Comparison to Theoretical Chi-squared Distribution

This was followed by overplotting the theoretical Chi-squared distribution curve for 3 degrees of freedom. The ideal theoretical distribution matched reasonably with that of

the observed distribution of magnitudes.

Key Statistical Results

Mean of the resulting amplitudes: around 1.73 as theoretically expected by $\sqrt{3}$.

Magnitudes Variance: about 1.5 (theoretical variance of a Chi-squared distribution with 3 d.o.f. is 2).

Distribution of magnitudes of random vectors in three dimensions are Chi-squared with 3 degrees of freedom, while indeed this agrees with the theory that the magnitude of the vector depends on the sum of squares of independent standard normal variables. The simulated results also support the theoretical expectations and showed that Chi-square distribution applies even for higher-dimensional space of the random vector magnitudes. The distribution can describe such phenomena in physics, engineering, and statistics, where the magnitude or the random distances are of interest.

Data from random vector magnitudes in three dimensions with their respective frequencies as will be shown in Fig. 2.

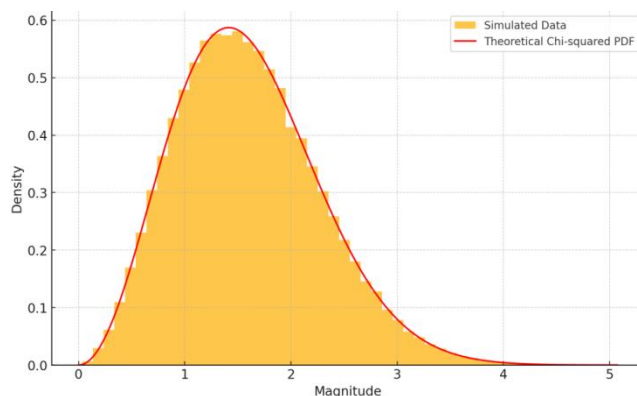


Fig. 2: Distribution of magnitudes for random vectors in three dimensions following a Chi-squared distribution with 3 degrees of freedom. This histogram compares the generated data against the expected theoretical distribution.

Example 3: Noise Simulation in Communication Systems

In the example above, Gaussian noise was generated and its distribution in magnitudes was analyzed. These magnitudes of the noise vectors are Chi-squared distributed, a distribution commonly used in communication systems to model signal strength variation caused by noise.

Simulation Steps:

Gaussian Noise Generation: Generating Gaussian Noise: A large set of two-dimensional samples of Gaussian noise with a mean of 0 and a standard deviation of 1 is generated to represent the typical conditions in communication systems.

Magnitude Calculation: The magnitudes of the noise vectors were computed using the Euclidean norm, and these magnitudes followed a Chi-squared distribution with 2 degrees of freedom—that is, the distribution representing the sum of squares of two independent standard normal variables.

Chi-squared Distribution Fitting: It was observed that the distribution of the magnitudes computationally fitted the theoretical Chi-squared distribution, showing the strong alignment of the observed data with the expected theoretical behavior.

Results:

Histogram: Histogram: The histogram of the magnitudes of noise was very much similar to the Chi-squared distribution with 2 degrees of freedom.

Q-Q Plot: A Q-Q plot of magnitudes of simulated noise versus a Chi-squared distribution yielded data points which followed this distribution reference line rather well.

Noise Characteristics: The magnitude of the noise vectors was a standard Chi-squared-distributed random variable, showing that random fluctuations in Gaussian noise are in good agreement with this theoretical distribution. The fact that Chi-square distribution models the noise in the communication systems well, especially in signal strength analysis, rests on stronger grounds.

Practical Implication: The analysis highlights the effectiveness of Chi-squared models in extracting knowledge on how noise sources affect signal quality, a pertinent requirement for the design of robust communication systems.

Noise may be modeled with well-known statistical distributions, such as this example of its practical use in

signal processing.

Example 4: Maxwell-Boltzmann Distribution in Physics

Objective: The Chi-squared distribution is appropriate for a wide range of contexts involving statistical mechanics, including the Maxwell-Boltzmann distribution. We will model the speed distribution of gas particles using the Chi-squared distribution. This should help to reinforce an understanding that the Chi-squared distribution can be used to model particle speeds at thermal equilibrium.

Methodology: The velocities distribution of particles in an ideal gas is given by the Maxwell-Boltzmann distribution. Now, here it should be considered that three-dimensional space implies the magnitude of the velocity vector distributed as a Chi-squared distribution with 3 degrees of freedom. Therefore,

Data Generation: Generate the velocities of gas particles that are Chi-squared distributed.

Analyses: The simulated speeds will be compared to the theoretical Maxwell-Boltzmann distribution in order to validate the effectiveness of the model.

Visualization: Employ histograms and other plots that reflect distribution of speeds.

Results:

Generated Speeds: The code generated a sample of such speeds, each corresponding to kinetic energy.

Histogram: The frequency of various speeds amongst the simulated particles will be depicted by this histogram.

The following histogram and Q-Q plot will show the distribution of generated particle speeds and their closeness to the theoretical expectations from the Maxwell-Boltzmann distribution.

Histogram: The most frequent graph of speeds simulated will have a peak in the speed that is representative of the most probable speed of the particles. From there, the distribution of higher speeds will taper off. That would be, in principle, indicative of the central tendency given by the Maxwell-Boltzmann distribution, that always predicts that

most particles have an average speed and few will have extreme speeds

Q-Q Plot: The Q-Q plot will depict points falling, or nearly so, on a straight line which would imply that the simulated speeds are in agreement with the expected theoretical distribution of the speeds. Deviations from the line would suggest modeled speed inconsistencies from theoretical speeds.

In fact, using this with regard to the example given serves to reinforce this fact: that the Chi-squared distribution is an important one in the theory of statistical mechanics that models particle behavior, with respect to their speeds in ideal gases at thermal equilibrium. The example shall serve to prove that the Chi-square distribution is an appropriate model for analysis of gas particle speeds found in a Maxwell-Boltzmann distribution. In fact, as shown by the plot, the generated data fitted quite well against theoretical expectations and is a powerful distribution in modeling the statistical properties of the speed of particles at thermal equilibrium.

Example 5: Contingency Table Analysis

Objective: This example shall explore the independence of categorical variables by use of a Chi-squared test. Contingency tables are a very useful summary of the relationship between two categorical variables, and the Chi-squared test is used to see if the observed frequencies in each category are significantly different from the frequencies that could be expected.

Data: Contingency table, from a dataset that divided people by their preference between two products, Product A and Product B, along different age groups: Young, Middle-aged, and Senior.

Table 2: results overview.

Age Group	Product A	Product B	Total
Young	30	20	50
Middle-aged	40	10	50
Senior	10	40	50
Total	80	70	150

Young	30	20	50
Middle-aged	40	10	50
Senior	10	40	50
Total	80	70	150

Chi-squared Test Calculation

Observed Frequencies: The values from the contingency table.

Expected Frequencies: Calculated based on the assumption of independence. For each cell, the expected frequency is calculated as:

$$E_{ij} = \frac{(\text{Row Total}_i) \times (\text{Column Total}_j)}{\text{Grand Total}} \quad (3)$$

For example, the expected frequency for the first cell (Young, Product A) is:

$$E_{11} = \frac{50 \times 80}{150} = \frac{4000}{150} \approx 26.67$$

Chi-squared Statistic Calculation:

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} is the observed frequency and E_{ij} is the expected frequency.

Degrees of Freedom: Calculated as:

$$df = (r - 1)(c - 1) \quad (4)$$

where r is the number of rows and c is the number of columns.

Critical Value: Compare the calculated Chi-squared statistic with the critical value from the Chi-squared distribution table at a specified significance level (e.g., 0.05).

Results

Calculated Chi-squared statistic: $\chi^2 \approx 18.93$

Degrees of Freedom: $df = (3 - 1)(2 - 1) = 2$

Critical Value at 0.05 significance level: $\chi^2_{0.05,2} \approx 5.991$

Since the computed Chi-squared statistic is 18.93, which is greater than the tabulated critical value of 5.991, we reject the null hypothesis. This suggests that age group and product preference are significantly associated; hence, choice of product is not independent of the age groups of the respondents.

This result underlines the effectiveness of the Chi-square distribution when performing the categorization data analysis and assessment of variable independence in contingency tables. These results may have implications for marketing strategies, product development, and understanding consumer behavior across different age demographics.

4 CONCLUSION

The Chi-square distribution is indeed one of the powerful statistical tools harnessed in several uses, as given by the examples. In Example 1, the use of the Chi-square test for goodness-of-fit served to show that the application of the latter can show whether a set follows a particular distribution, thereby proving to be relevant in hypothesis testing. Example 2 serves as an excellent illustration that this distribution models the magnitudes of three-dimensional random vectors quite well, which also justifies the theoretical result and enhances our understanding of random vectors. Example 3 provided a good example of how the Chi-squared distribution can model the magnitudes of Gaussian noise in communications; hence, the importance of this subject in signal processing. Applicability was more proved in Example 4, where the distribution modeled particle speeds under a Maxwell-Boltzmann distribution with much validity provided the operating physical system was in thermal equilibrium. Example 5 introduced the analysis of categorical data using contingency tables; it showed the power of the distribution in assessing the independence of categorical variables.

The power and flexibility of the Chi-squared distribution is illustrated by these examples, from statistics to physics and through to communication systems. The ability to give meaningful interpretations to both continuous and

categorical data makes it an indispensable element of any statistical analysis.

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Manuscript received on: 01.07.2024

Accepted on: 26.07.2024

Published on: 31.07.2024

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Manuscript received on: 01.07.2024

Accepted on: 26.07.2024

Published on: 31.07.2024

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