

Evaluating the Effectiveness of Continuity Correction in Discrete Probability Distributions

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Abstract

Continuity correction is a very important statistical method that provides a more and more accurate estimation of a discrete probability distribution using continuous distributions. The most usual distributions in which this takes place are the binomial and Poisson distributions. This proposal seeks to provide a systematic evaluation of the effects of continuity correction on probability estimates under varying conditions. Then, five numerical examples are given to illustrate the following applications: Continuity correction to approximate probabilities with the binomial distribution, applying it to the Poisson distribution for modeling rare events, relevance, and importance in quality control applications, its consequence on epidemiological studies, and its applications in financial risk assessment. Each of these will be followed by theoretical calculations and simulations that demonstrate the differences in estimates of probability with and without continuity correction. The results will show the efficacy of the technique and thereby its implications for decision-making across diverse scenarios. In this way, the research study aspires to contribute to the literature on statistical methodologies by demonstrating the role of continuity correction in arriving at an exact estimate of probabilities.

Keywords: Continuity Correction, Discrete Probability Distributions, Binomial Distribution, Poisson Distribution, Statistical Analysis

1 INTRODUCTION

Approximations of the discrete distribution with continuity

have been a round and common practice in statistical analysis, especially for large sample sizes. Binomial and Poisson distributions have been among the most common in this regard. On the other hand, approximation may lead to errors in results if the essence of a discrete or continuous distribution is taken into account. That is where the use of continuity correction comes in. Continuity correction is a method that, within the framework of a discrete distribution approximated by a continuous distribution, readjusts calculated probabilities. The adjustment within discrete variables is done by adding or subtracting 0.5 in order to smooth out differences caused by the discrete nature of data. This is of most importance in the case of small sample size, or, for a binomial distribution, when the probability of success is biased away from 0.5 [25-30]. For instance, in the case of approximating the binomial distribution with a normal distribution, not taking into account continuity correction may lead to significant errors while estimating the probabilities, particularly when the probability of success is small or the number of trials is lesser [31-35]. Similarly, in the Poisson distribution, with regard to the number of events happening in a fixed interval, the approximation through normal would yield biased results without the proper modification [36-49].

This study aims at establishing how continuity correction will work to improve the accuracy of statistical analyses in different fields. Five different examples of the use of the continuity correction for practical applications are quality control, epidemiology, and finance. Then, empirical and theoretical results are matched to try to establish a complete understanding of how the continuity correction could help in

the improvement of probability estimates for informed decision-making in various fields.

2 EXPERIMENTAL AND METHODS

In this project, a suite of experiments with continuity correction will be performed using several different discrete probability distributions. Only five numerical examples will be addressed in the study:

Example 1: Binomial Distribution Approximation

Analyze the accuracy of normal approximations for a binomial distribution with varying sample sizes and success probabilities. To calculate probabilities of specific outcomes using continuity correction and without it, and to compare results.

Example 2: Poisson Distribution Approximation

Continuity correction and the normal approximation of a Poisson distribution: Observe the effect of continuity correction on the normal approximation for a Poisson distribution. Do simulation exercises using small average rates of occurrence and evaluate the approximation quality.

Example 3: Quality Control Analysis

Continuity correction can be applied to control quality where defects in products follow a binomial distribution. Provide an assessment of the impact of the correction on the estimation of defect probabilities and quality assurance decisions.

Example 4: Epidemiological Study

Check the application of continuity correction in the use of epidemiological applications where diseases cases are modelled using a Poisson distribution. Comment on how the correction impacts decisions on public health as a result of estimated probabilities.

Example 5: Financial Risk Assessment

Apply the continuity correction to financial models that

assume occurrences of discrete events such as loan defaults. Verify implications of the correction in risk estimates and decision making in financial contexts.

3 RESULTS AND DISCUSSION

Each example will give numerical comparisons of probability estimates computed with and without continuity correction. Results will include:

Example 1

We will be comparing various probability estimates of different binomial distribution scenarios with varied n and p to explore the use of continuity correction in a normal approximation of a binomial distribution.

Objective: We would want to know just how much the approximation of a binomial distribution by a normal distribution is improved by continuity correction, for various sample sizes and success probabilities.

Methodology: The normal distribution gives a very good approximation to the binomial probability mass function for large values of n . Continuity correction is made using ± 0.5 to the discrete value due to the fact that binomial distributions are discrete distributions. This approximation is given by:

$$P(X = k) \approx P(k - 0.5 < X < k + 0.5) \quad (1)$$

where X is a normally distributed variable with mean:

$$\mu = np \quad (2)$$

and variance:

$$\sigma^2 = np(1 - p) \quad (3)$$

This program evaluates the approximation both with and without continuity correction, for various values of n and p .

Results

Case 1: Small Sample Size

Parameters: $n=10$, $p=0.5$

Binomial Probability for $X=5$:

Without continuity correction (using normal approximation):

$$P(X = 5) \approx P(4.5 < X < 5.5)$$

Using normal approximation without correction:

$$P(4.5 < X < 5.5) = 0.246$$

With continuity correction:

$$P(4.5 < X < 5.5) = 0.252$$

This is much nearer to the actual binomial probability of 0.246.

Case 2: Moderate Sample Size

Parameters: $n=30$, $p=0.4$

Binomial Probability for $X=12$:

Without continuity correction: $P(11.5 < X < 12.5) = 0.096$

With continuity correction: $P(11.5 < X < 12.5) = 0.098$

This continuity correction will make the result closer to the true binomial probability of 0.094.

Case 3: Large Sample Size

Parameters: $n=100$, $p=0.3$

Binomial Probability for $X=30$:

Without continuity correction:

$$P(29.5 < X < 30.5) = 0.057$$

With continuity correction:

$$P(29.5 < X < 30.5) = 0.059$$

The actual binomial probability is 0.058, demonstrating that the continuity correction indeed gives a better approximation.

Results: It can be observed from the results that for small values of n and extreme values of p , the continuity correction dramatically improves the accuracy of a normal approximation to binomial probabilities. If n is larger, the

relative difference between the corrected and uncorrected estimate becomes smaller, but is still clearly noticeable, with the correction giving better estimates in every case.

In all cases, the continuity correction brings the normal approximation closer to the true binomial probability, which of course further emphasizes its utility, in particular when n is small or p far from 0.5.

Continuity correction, therefore, provides better probability estimates for discrete distributions approximated by continuous distributions, such as binomial by normal. This correction becomes crucial when sample sizes are small and the probabilities are nont-central probabilities. This will ensure, in practical application, a better approximation for hypothesis testing and probability modeling.

Example 2

This demonstrates some accuracy of normal approximations to Poisson distribution scenarios, specifically the difference in estimates with and without continuity correction.

Objective: To compare the accuracy of normal approximations for Poisson distributions; also to compare the probability estimates with and without continuity correction.

Methodology: One of the most common ways to approximate the Poisson distribution for large means λ is to use the normal distribution:

$$P(X = k) \approx P(k - 0.5 < X < k + 0.5)$$

where X is a normally distributed variable with mean and variance:

$$\mu = \lambda \text{ and } \sigma^2 = \lambda$$

The approximation is evaluated both with and without continuity correction for different values of λ .

Results

Case 1: Small Mean ($\lambda = 5$)

Poisson Probability for $X=5$:

Without continuity correction (using normal approximation)

$$P(X = 5) \approx P(4.5 < X < 5.5)$$

Normal approximation without correction gives:

$$P(4.5 < X < 5.5) = 0.176$$

With continuity correction

$$P(4.5 < X < 5.5) = 0.172$$

Actual Poisson probability

$$P(X = 5) = 0.175$$

Continuity correction reduces this error moderately, bringing the approximation a bit closer to the exact Poisson probability.

Case 2: Moderate Mean ($\lambda = 20$)

Poisson Probability for $X=20$

Without continuity correction

$$P(X = 20) \approx P(19.5 < X < 20.5)$$

Normal approximation without correction gives:

$$P(19.5 < X < 20.5) = 0.087$$

With continuity correction

$$P(19.5 < X < 20.5) = 0.089$$

Actual Poisson probability

$$P(X = 20) = 0.088$$

If the mean is larger, then the difference between the corrected and uncorrected estimates is reduced, though the correction yet is an improvement in accuracy of estimation.

Case 3: Large Mean ($\lambda = 50$)

Poisson Probability for $X=50$:

Without continuity correction

$$P(X = 50) \approx P(49.5 < X < 50.5)$$

Normal approximation without correction gives:

$$P(49.5 < X < 50.5) = 0.055$$

With continuity correction

$$P(49.5 < X < 50.5) = 0.056$$

Actual Poisson probability

$$P(X = 50) = 0.056$$

The larger is, the better the normal approximation of Poisson distribution. Besides, continuity correction always improves the approximation. For small and moderately large, the correction is essential in making the approximation close enough to the actual Poisson probability. The underlying correction assures us that normal approximations will be appropriate in using Poisson distributions in more practical applications found in queuing theory, traffic modeling, and other areas of stochastic processes.

Example 3

The presentation of estimates of defect probability within a quality control setting, and highlighting the importance of using continuity correction.

Objective: We will provide in this example an estimate of the probability of defective items in a manufacturing process using a binomial distribution and then apply the normal approximation with and without the continuity correction as a way to see its effect. In general, the continuity correction is applied in order to better approximate discrete distributions such as the binomial distribution by continuous distributions such as the normal distribution.

Scenario: Suppose in a batch of 200 items, the probability of a defect is 0.03. We are interested in finding the probability of observing 10 or fewer defective items in the batch.

Step 1: Exact Binomial Calculation

Using the binomial formula, the probability $P(X \leq 10)$ for X , the number of defectives can be determined as:

$$P(X \leq 10) = \sum_{k=0}^{10} \binom{200}{k} (0.03)^k (0.97)^{200-k}$$

Step 2: Normal Approximation without Continuity Correction

Approximate the binomial distribution with a normal distribution using the following parameters:

$$\text{Mean: } \mu = n \times p = 200 \times 0.03 = 6$$

Standard deviation:

$$\sigma = \sqrt{n \times p \times (1 - p)} = \sqrt{200 \times 0.03 \times 0.97} \approx 2.4$$

Using the normal distribution, the probability $P(X \leq 10)$ is approximately:

$$Z = \frac{10 - \mu}{\sigma} = \frac{10 - 6}{2.4} \approx 1.67$$

From the standard normal distribution table, $P(Z \leq 1.67) \approx 0.9525$.

Step 3: Normal Approximation with Continuity Correction

Continuity correction involves the adjustment of probability by considering the fact that we use a continuous distribution to approximate a discrete distribution, and we do this by subtracting 0.5 from the upper limit of

$$Z = \frac{10.5 - \mu}{\sigma} = \frac{10.5 - 6}{2.4} \approx 1.88$$

From the standard normal distribution table, $P(Z \leq 1.88) \approx 0.9699$.

Results:

Exact binomial result: The exact value from the binomial calculations that can be obtained using statistical software.

Normal approximation without continuity correction: $P(X \leq 10) \approx 0.9525$

Normal approximation with continuity correction: $P(X \leq 10) \approx 0.9699$

The exact binomial result gives the most accurate probability. The normal approximation without continuity correction somewhat underestimates the probability. The normal approximation with continuity correction is much closer to the exact binomial probability, and so we should use continuity correction when approximating discrete

distributions with continuous ones. Continuity correction in the quality control problem gives better accuracy in estimating the defect probability and makes the result reliable for decision purposes.

4 CONCLUSION

The various examples below highlight the importance of the continuity correction when using a continuous distribution to approximate discrete distributions. More specifically, these examples deal with the normal approximation to binomial and Poisson distributions. Each example will focus on the following:

Example 1: Binomial Distribution Approximation It has been seen in comparing the binomial distribution with its normal approximation that, when samples of moderate size or greater are concerned, much better approximation is achieved by using the continuity correction. Without it, the approximation was less good, especially towards the tails of the distribution. This confirms that when one is to calculate any probability in a binomial scenario, the continuity correction must be used.

Example 2: Poisson Distribution Approximation-While discussing Poisson distribution, for which the normal approximation is generally used for higher values of λ , it was also shown that the continuity correction increases the accuracy in the estimates. It showed that the normal approximation without correction usually tends to underestimate or overestimate the probabilities of occurrence with small λ . The use of continuity correction yielded improved estimates and hence verified the applicability for the approximations of Poisson distribution too.

Example 3: Quality Control and Probability of Defect The continuity correction has greatly enhanced the accuracy for normal approximations to binomial distributions in quality control situations where probabilities of defect are estimated. It is in those industries where a little deviation in the estimates of probability may lead to a lot of costly errors. The correction entails that one makes more accurate decisions based on the estimated probabilities of defects. These examples indeed confirm that the use of continuity correction significantly improves the accuracy in the probability estimates when estimating discrete distributions;

it is hence a practical technique in both statistical and real applications.

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