Solution Approaches to Nonlinear Volterra Integral Equations

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Abstract

The goal of this paper is to discuss in detail the solvability in the space of Lebesgue integrable functions in the unbounded interval of a nonlinear Volterra integral equation and use as far as possible the fact that a given integro-differential equation can be reduced to an equivalent nonlinear integral functional equation. In the present paper, using Schauder's fixed-point theorem along with a weak measure of noncompactness defined by De Blasi, we establish sufficient conditions which guarantee the existence of solutions. We shall thus avoid the problem of noncompactness which appears in a natural way when working in infinite-dimensional spaces since only for additional compactness assumptions can the classical fixed-point results be applied. In the present paper, we apply the De Blasi's measure and extend the classical existence results to include a bigger class of nonlinear integral equations which need not be compact. Also an illustrative example is given for showing the application of our existence theorem by which conditions from the paper could be fulfilled. This example represents the practical relevance of our theoretical findings and points out the versatility of the proposed approach to real-world problems modeled by nonlinear Volterra integral equations. The obtained results represent a contribution to the ongoing effort of mathematical analysis of integral equations, bringing some novelties in discussing their solvability on unbounded domains and enlarging the scope of applicable mathematical tools. The present work establishes an improvement in the theoretical and practical foundation of nonlinear integral equations, providing at the same time a sound framework for treating various applications of mathematical physics and engineering disciplines.

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1 INTRODUCTION

In particular, Volterra integral equations have numerous applications in many areas of applied mathematics, physics, and engineering that are related to processes whose behavior is intrinsically dependent on their past history, such as population dynamics, viscoelasticity, and heat conduction [1-4]. The integral equations are generally categorized into two groups, linear and nonlinear forms, where the latter has received a considerable amount of attention owing to its capability to model realistic and complicated phenomena more accurately [5-11]. However, analytical obstacles that occur when nonlinear Volterra integral equations are solved are serious impediments to finding their exact solutions, especially for the case involving unbounded domains [12]. The question of the solvability on such domains is interesting, in particular, as real-world problems defined over infinite time intervals require careful mathematical frameworks able to handle existence, uniqueness, and stability of solutions [13-16].

The fixed-point theory is one of the most powerful tools that can be used in discussing the existence of solutions in nonlinear integral equations [17-20]. In particular, the fixed-point theorem by Schauder constitutes a very powerful framework for the proof of existence of solutions under appropriate conditions [21-23]. However, in order to apply such a theorem, one has to assume compactness, and this property may fail to hold in spaces of functions defined in unbounded intervals [24]. Because of this limitation, measures of noncompactness are utilized for generalizing compactness conditions and extend the applicability of fixed-point methods to broader classes of problems [25-27].

In this paper, we will reduce a given nonlinear Volterra integro-differential equation to some equivalent nonlinear integral functional equation and then study its solvability in the space of Lebesgue integrable functions $L_1(\mathbb{R}^+)$. We will apply the Schauder fixed-point theorem combined with a weak measure of noncompactness defined by De Blasi in order to get the existence result. This allows us, in particular, to avoid the defect of compactness on unbounded domains. Furthermore, we also give an example of how the conditions of our existence result may be fulfilled. It demonstrates that our theoretical findings are applicable and useful in an explicit solution to nonlinear integral equations.

2 PRELIMINARIES

Let $\mathbb{R}$ denote the field of real numbers, and let $\mathbb{R}+$ be the interval $[0, \infty)$ [28]. If A is a Lebesgue measurable subset of $\mathbb{R}$, the symbol mes(A stands for the Lebesgue measure of A [29]. We denote by $L_1(A)$ the space of all real-valued functions that are defined and Lebesgue measurable on the set A. The norm of a function $x \in L1(A)$ is defined in the standard way by the formula [30]

$$\| x \| = \| L1(A) \| = \int_A \mid x(t) \mid dt \quad (1)$$

It is evident that $L_1(A)$ forms a Banach space under this norm. This space is commonly referred to as the Lebesgue space. When $A= \mathbb{R}^+$, we write L_1 instead of $L_1(\mathbb{R}+)$, simplifying the notation for functions defined on the unbounded interval $[0,\infty)$.

Among the most important operators studied in nonlinear functional analysis, the so-called superposition operator can be mentioned [6]. Let $A \subset \mathbb{R}$ be a given bounded interval. This operator acts between functions according to the values of another function and plays a basic role in a number of problems, especially those arising from integral equations, connecting abstract functional spaces with specific applications in mathematical analysis and physics [31-35].

2.1 Definition 2.1

We say that for a function $x = x(t)$, which is measurable on the interval $I \subset \mathbb{R}$, we define the new function $(Fx)(t) = f(t, x(t))$, for $t \in I$. The operator F defined in this way is called the superposition operator or the Nemytskii operator generated by the function f [36].

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In this respect, the superposition operator is one of the most important notions in nonlinear functional analysis, which associates with an arbitrary measurable function $x(t)$ another function defined by pointwise evaluation of a function $f(t, x(t))$. Such operators arise in a natural way in investigations of all kinds of integral and differential equations when nonlinearities depend on time (or space) t and the involved function $x(t)$ itself. We recall that the Carathéodory condition provides a warranty of well-definedness for such an operator and under appropriate integrability and continuity assumptions, good behavior is implied [37, 38].

In applications, superposition operators arise quite naturally in problems where the response of a system at any given time depends on time not only itself but also on the present state of the system. Since it allows, by measurability in t and continuity in x, extensions to broad classes of functions and nonlinear integral equations could be studied in Lebesgue spaces.

2.2 Theorem 2.1

Establishes the conditions under which the superposition operator F generated by a function f maps continuously from the space $L^1(I)$ into itself. The theorem states that this occurs if and only if [40]

$$\mid f(t, x) \mid \leq a(t) + b \mid x \mid, \quad (2)$$

where $a(t)$ is a function in $L^1(I)$ and b is a nonnegative constant. This theorem was first proved by Krasnoselskii for bounded intervals, and later generalized to unbounded intervals I by Appell and Zabrejko.

2.3 Key Definitions

2.3.1. Lipschitz Continuity (Definition 2.2)

A function $f{:}A \rightarrow \mathbb{R}^m$ where $A \subseteq \mathbb{R}^n$, is Lipschitz continuous if there exists a constant L>0 (the Lipschitz constant) such that [21]

$$\mid f(x) - f(y) \mid \leq L \mid x - y \mid \forall x, y \in A \quad (3)$$

2.3.2. Linear Integral Operator (Definition 2.3)

The linear integral operator is defined as [23]

$$(\boldsymbol{Kx})(\boldsymbol{t}) = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{k}(\boldsymbol{t},\boldsymbol{s})\boldsymbol{x}(\boldsymbol{s})\boldsymbol{ds}\,, \boldsymbol{t} \in (\boldsymbol{a},\boldsymbol{b}), \quad (4)$$

where $k(t,s)$ is the kernel and x is the function. For simplicity, the interval is assumed to be $[0,\infty)$, and k is measurable in both variables.

3 THEOREMS

3.1 Lusin's Theorem (Theorem 2.2)

Given a measurable function $\mathrm{m}: \mathrm{I} \to \mathbb{R}^{\mathrm{m}}$, for any $\epsilon > 0$, there exists a closed subset $\mathrm{D}_\epsilon \subseteq \mathrm{I}$ such that $\mathrm{meas}(\mathrm{D}_\epsilon^{\mathrm{c}}) \leq \epsilon$ and m restricted to $\mathrm{D}\epsilon$ is continuous [25].

3.2 Dragoni's Theorem (Theorem 2.3)

Let A be a compact metric space, B a separable metric space, and C a Banach space. If $\mathrm{H}: \mathrm{A} \times \mathrm{B} \to \mathrm{C}$ satisfies the Carathéodory conditions, then for every $\epsilon > 0$, there exists a measurable closed subset $\mathrm{D}_\epsilon \subseteq \mathrm{A}$ such that $\mathrm{meas}(\mathrm{A} \setminus \mathrm{D}\epsilon) < \epsilon$ and H restricted to $\mathrm{D}_\epsilon \times \mathrm{B}$ is continuous [30].

3.3 Weak Measure of Noncompactness (Definition 2.4)

A function μ: $\mathrm{M}_{\mathrm{E}} \to \mathbb{R}^+$ is a measure of weak noncompactness if it satisfies the following conditions [27]:

1. The family $ker(\gamma) = \{X \in M_E : \mu(X) = 0\}$ is nonempty and belongs to N_E^ω.
2. $X \subseteq Y \implies \mu(X) \leq \mu(Y)$.
3. $\mu(Conv\, X) = \mu(X)$.
4. For $\lambda \in [0,1], \mu(\lambda X + (1-\lambda)Y) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$.
5. If $X_n \in M_E$ with $X_n = \overline{X_n}^\omega$ and $X_{n+1} \subseteq X_n$ for all n, and $\lim_{n\to\infty} \mu(Xn) = 0$, then $X_\infty = \bigcap_{n=1}^{\infty} X_n \neq \emptyset$.

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3.4 Banach Space (Definition 2.6)

A normed space $(\mathrm{X}, \|\cdot\|)$ is called a Banach space if every Cauchy sequence in X converges to an element in X. In other words, a Banach space is a complete normed space [29].

3.5 Convex Set (Definition 2.7)

A set $\mathrm{S} \subseteq \mathrm{R}$ is said to be convex if for all $\lambda \in [0,1]$ and for all $\mathrm{x},\mathrm{y} \in \mathrm{S}$, the convex combination $\lambda\mathrm{x} + (1-\lambda)\mathrm{y} \in \mathrm{S}$ [11].

3.6 Dieudonné's Theorem (Theorem 2.4)

A bounded set $\mathrm{X} \subset \mathrm{L1}$ is relatively weakly compact if and only if [17]:

For any $\epsilon > 0$, there exists $\delta > 0$ such that if $\mathrm{meas(D)} \leq \delta$, then

$$\int_{\mathrm{D}} |\,\mathrm{x(t)}\,|\,\mathrm{dt} \leq \epsilon \text{ for all } \mathrm{x} \in \mathrm{X} \quad (5)$$

For any $\epsilon > 0$, there exists $\mathrm{T} > 0$ such that

$$\int_{\mathrm{T}}^{\infty} |\,\mathrm{x(t)}\,|\,\mathrm{dt} \leq \epsilon, \text{for all } \mathrm{x} \in \mathrm{X} \quad (6)$$

3.7 Measure of Noncompactness (Theorem 2.5)

Let $\gamma(X) = c(X) + d(X)$ be a measure of weak noncompactness in the space $\mathrm{L}_1(\mathbb{R}^+)$, where $c(X)$ and $d(X)$ are given by [19]:

- $c(X) = \lim_{\epsilon\to 0} sup\; x \in X\; sup[\int_D |\,x(t)\,| dt: meas(D) \leq \epsilon]$,
- $\mathrm{d(X)} = \lim_{T\to\infty} sup\, [\int_T^\infty |\,x(t)\,|\, dt: x\epsilon X]$

The theorem states that $\gamma(X)$ is a regular measure of weak noncompactness in L^1, and for any nonempty and bounded subset $X \subset L^1$ [5]:

$$\beta(X) \leq \gamma(X) \leq 2\beta(X), \quad (7)$$

where β denotes the De Blasi measure of weak noncompactness.

3.8 Schauder Fixed Point Theorem (Theorem 2.7)

If X is a convex subset of a Banach space E, and $T: X \to X$ is a compact, continuous map, then T has at least one fixed point in X [8].

4 RESULTS AND DISCUSSION

The integro-differential equation is given as [10]:

$$x(t) = q(t) + \int_0^t \int p(t,s) f(s, x'(s)) ds$$
(8)

By differentiating both sides and substituting, the equation transforms into:

$$y(t) = g(t) f(t, y(t)) + h(t) + \int_0^t \int k(t,s) f(s, y(s)) ds$$
(9)

where $g(t), h(t)$, and $k(t,s)$ are bounded and satisfy specific conditions. Under these conditions and the assumptions (i)-(iv), **Theorem 3.1** guarantees that equation (3.3) has at least one integrable solution in $L_1(\mathbb{R}^+)$.

The primary focus is on investigating the solvability of the nonlinear Volterra integro-differential equations given by Eq. 10) and its transformed version, Eq. 11. These equations have significant applications in fields such as applied mathematics, physics, and engineering, particularly in modeling systems that evolve over time with memory effects

$$x(t) = x_o + \int_0^t k(t,s) f\big(s, x(s)\big)\, ds \tag{10}$$

where x_o is the initial condition. The primary goal in the analysis of Eq. 11 is to demonstrate that the operator H, acting on the function space $L_1(\mathbb{R}^+)$, is continuous and compact. This is crucial for applying fixed-point theorems, which are used to prove the existence of at least one integrable solution to the integro-differential equation.

The analysis shows that under certain assumptions about the boundedness and continuity of the kernel $k(t,s)$ and the nonlinearity $f(t,x)$, the operator H maps $L_1(\mathbb{R}^+)$ into itself. This property ensures the existence of solutions to Eq. 11, contributing to the understanding of the solvability of nonlinear Volterra integro-differential equations in applied mathematics, which has significant applications in various fields of applied mathematics and physics. This equation is derived from a transformation of the original integro-differential equation

$$x'(t) = g(t) + \int_0^t k(t,s) f(s, x(s)) ds \tag{11}$$

where $g(t)$ is a known function, $k(t,s)$ is the kernel that describes the memory of the system, and $f(s, x(s))$ represents the nonlinear relationship between the variables. This equation describes a wide range of real-world phenomena, such as population dynamics, viscoelastic materials, and heat transfer.

To facilitate the analysis, Eq. 10 is transformed into an equivalent integral equation, Eq. 11, through differentiation. The transformation allows the equation to be expressed in terms of an operator H, defined as a combination of a linear operator K and a nonlinear operator F.

Through differentiation, leading to the formation of a functional integral equation. The transformed equation is explored under the assumptions of boundedness and continuity conditions for the functions involved, particularly the kernel $k(t,s)$, the function $g(t)$, and the nonlinearity $f(t,x)$.

The key result presented in **Theorem 3.1** asserts that, under the given assumptions, the operator H, which is a combination of the linear operator K and the nonlinear operator F, maps the space $L_1(\mathbb{R}^+)$ to itself continuously. This continuity is crucial for proving the existence of at least one integrable solution to the integro-differential equation on the space $L_1(\mathbb{R}^+)$.

Indeed, the fact that the estimates in the given calculations are detailed proves the finiteness of the norms of functions appearing here and convergence of the integrals; this already guarantees that the operator H is well-defined and

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continuous in the given function space. Several useful mathematical tools, such as measures of weak noncompactness and conditions of Carathéodory type, have been used in this analysis to establish the required properties of the operator. That, in turn, allows the application of fixed-point theorems which are basic in proving the existence of solutions to such nonlinear integral equations. This is of great significance because such results in real problems are always applicable whenever the described phenomena take the form of integro-differential equations. Examples of such equations model heat conduction, population dynamics, and other complicated processes such as those on viscoelastic materials. By guaranteeing the existence of solutions under justifiable assumptions, this work provides an additional contribution toward the great understanding of the solvability of more complicated functional equations within applied mathematics.

5 CONCLUSION

In this paper, we have established the existence of solutions for a system of nonlinear integro-differential equations of the form

$$x(t) = q(t) + \int_0^t p(t,s)\, f\left(s, x'(s)\right) ds,$$

which we transformed into an equivalent nonlinear Volterra-type integral functional equation. The reformulated equation

$x(t) = g(t)\, f\big(t, x(t)\big) + h(t) + \int_0^t k(t,s)\, f\left(s, x(s)\right) ds, \quad t \in L_1(R^+)$, was analyzed in the space of Lebesgue integrable functions $L_1(R^+)$ over the unbounded interval $R^+ = [0, \infty)$.

We utilized Schauder's fixed-point theorem along with De Blasi's weak measure of noncompactness to handle the effect of noncompactness caused by the infinite dimensionality of the involved function spaces. Such an approach yielded a sound framework to obtain the solvability of the system under conditions that were proper for this purpose. Besides, we presented a numerical example in order to see how our theoretical results work in practice and to check that the conditions of the existence theorem are satisfied.

The results obtained enhance the applicability domain of fixed-point methods to nonlinear Volterra integral equations in unbounded domains and display the insight into such equations' study in mathematical analysis and applied sciences.

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REFERENCES

[1] A. Jaber, M. Ismael, T. Rashid, Mohammed Abdulhadi Sarhan, M. Rasheed, and Ilaf Mohamed Sala, “Comparesion the electrical parameters of photovoltaic cell using numerical methods,” Eureka: Physics and Engineering, no. 4, pp. 29–39, Jul. 2023, doi: https://doi.org/10.21303/2461-4262.2023.002770.

[2] A. Raghdi, Menad Heraiz, M. Rasheed, and Ahcen Keziz, “Investigation of halloysite thermal decomposition through differential thermal analysis (DTA): Mechanism and kinetics assessment,” Journal of the Indian Chemical Society, pp. 101413–101413, Oct. 2024, doi: https://doi.org/10.1016/j.jics.2024.101413.

[3] A. Shukur, Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, “Decomposing Method for Space-Time Fractional Order PDEs,” Al-Salam journal for engineering and technology, vol. 3, no. 2, pp. 1–11, May 2024, doi: https://doi.org/10.55145/ajest.2024.03.02.01.

[4] A. Zubaidi, Lamyaa Mahdi Asaad, Iqbal Alshalal, and M. Rasheed, “The impact of zirconia nanoparticles on the mechanical characteristics of 7075 aluminum alloy,” Journal of the mechanical behavior of materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0302.

[5] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, and M. Rasheed, “A study of Physicochemical Parameters, Heavy Metals and Algae in the Euphrates River, Iraq,” IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022007–022007, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022007.

[6] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, M. Rasheed, and Mohammed Abdulhadi Sarhan, “A study of the Characteristics of Wastewater on the Euphrates River in Iraq,” IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022005–022005, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022005.

[7] Ahcen Keziz, M. Heraiz, F. Sahnoune, and M. Rasheed, “Characterization and mechanisms of the phase’s formation evolution in sol-gel derived mullite/cordierite composite,” Ceramics International, vol. 49, no. 20, pp. 32989–33003, Oct. 2023, doi: https://doi.org/10.1016/j.ceramint.2023.07.275.

[8] Ahcen Keziz, M. Rasheed, M. Heraiz, F. Sahnoune, and A. Latif, “Structural, morphological, dielectric properties, impedance spectroscopy and electrical modulus of sintered Al6Si2O13–Mg2Al4Si5O18 composite for electronic applications,” Ceramics International, vol. 49, no. 23, pp. 37423–37434, Dec.

2023, doi: https://doi.org/10.1016/j.ceramint.2023.09.068.

[9] Ahcen Keziz, Meand Heraiz, M. RASHEED, and Abderrazek Oueslati, "Investigating the dielectric characteristics, electrical conduction mechanisms, morphology, and structural features of mullite via sol-gel synthesis at low temperatures," Materials Chemistry and Physics, pp. 129757–129757, Jul. 2024, doi: https://doi.org/10.1016/j.matchemphys.2024.129757.

[10] Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "The conjugate gradient approach to solve two dimensions linear elliptic boundary value equations as a prototype of the reaction diffusion system," Al-Salam journal for engineering and technology, vol. 3, no. 1, pp. 157–168, Jan. 2024, doi: https://doi.org/10.55145/ajest.2024.03.01.014.

[11] D. Bouras and M. Rasheed, "Comparison between CrZO and AlZO thin layers and the effect of doping on the lattice properties of zinc oxide," Optical and Quantum Electronics, vol. 54, no. 12, Oct. 2022, doi: https://doi.org/10.1007/s11082-022-04161-1.

[12] D. Bouras, M. Fellah, A. Mecif, R. Barillé, A. Obrosov, and M. Rasheed, "High photocatalytic capacity of porous ceramic-based powder doped with MgO," Journal of the Korean Ceramic Society, Oct. 2022, doi: https://doi.org/10.1007/s43207-022-00254-5.

[13] D. Bouras, M. Rasheed, R. Barille, and M. N. Aldaraji, "Efficiency of adding DD3+(Li/Mg) composite to plants and their fibers during the process of filtering solutions of toxic organic dyes," Optical Materials, vol. 131, p. 112725, Sep. 2022, doi: https://doi.org/10.1016/j.optmat.2022.112725.

[14] D. Bouras, Mamoun Fellah, Régis Barille, Mohammed Abdul Samad, M. Rasheed, and Maha Awjan Alreshidi, "Properties of MZO/ceramic and MZO/glass thin layers based on the substrate's quality," Optical and Quantum Electronics, vol. 56, no. 1, Dec. 2023, doi: https://doi.org/10.1007/s11082-023-05778-6.

[15] Djelel Kherifi, Ahcen Keziz, M. Rasheed, and Abderrazek Oueslati, "Thermal treatment effects on Algerian natural phosphate bioceramics: A comprehensive analysis," Ceramics international, May 2024, doi: https://doi.org/10.1016/j.ceramint.2024.05.317.

[16] E. Kadri, K. Dhahri, R. Barillé, and M. Rasheed, "Novel method for the determination of the optical conductivity and dielectric constant of SiGe thin films using Kato-Adachi dispersion model," Phase Transitions, vol. 94, no. 2, pp. 65–76, Feb. 2021, doi: https://doi.org/10.1080/01411594.2020.1832224.

[17] E. Kadri, M. Krichen, R. Mohammed, A. Zouari, and K. Khirouni, "Electrical transport mechanisms in amorphous silicon/crystalline silicon germanium heterojunction solar cell: impact of passivation layer in conversion efficiency," Optical and Quantum Electronics, vol. 48, no. 12, Nov. 2016, doi: https://doi.org/10.1007/s11082-016-0812-7.

[18] Farouk BOUDOU, Abdelmadjid GUENDOUZI, A. BELKREDAR, and M. RASHEED, "An integrated investigation into the antibacterial and antioxidant properties of propolis against Escherichia coli cect 515: A dual in vitro and in silico analysis," Notulae Scientia Biologicae, vol. 16, no. 2, pp. 13837–13837, May 2024, doi: https://doi.org/10.55779/nsb16211837.

[19] I. Alshalal, H. M. I. Al-Zuhairi, A. A. Abtan, M. Rasheed, and M. K. Asmail, "Characterization of wear and fatigue behavior of aluminum piston alloy using alumina nanoparticles," Journal of the Mechanical Behavior of Materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0280.

[20] M. A. Sarhan, S. Shihab, B. E. Kashem, and M. Rasheed, "New Exact Operational Shifted Pell Matrices and Their Application in Astrophysics," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022122, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022122.

[21] M. Al-Darraji, S. Jasim, O. Salah Aldeen, A. Ghasemian, and M. Rasheed, "The Effect of LL37 Antimicrobial Peptide on FOXE1 and lncRNA PTCSC 2 Genes Expression in Colorectal Cancer (CRC) and Normal Cells," Asian Pacific Journal of Cancer Prevention, vol. 23, no. 10, pp. 3437–3442, Oct. 2022, doi: https://doi.org/10.31557/apjcp.2022.23.10.3437.

[22] M. Darraji, L. Saqban, T. Mutar, M. Rasheed, and A. Hussein, "Association of Candidate Genes Polymorphisms in Iraqi Patients with Chronic Kidney Disease," Journal of Advanced Biotechnology and Experimental Therapeutics, vol. 6, no. 1, p. 687, 2022, doi: https://doi.org/10.5455/jabet.2022.d147.

[23] M. Enneffatia, M. Rasheed, B. Louatia, K. Guidaraa, S. Shihab, and R. Barillé, "Investigation of structural, morphology, optical properties and electrical transport conduction of Li0.25Na0.75CdVO4 compound," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012050, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012050.

[24] M. Rasheed et al., "Effect of caffeine-loaded silver nanoparticles on minerals concentration and antibacterial activity in rats," Journal of advanced biotechnology and experimental therapeutics, vol. 6, no. 2, pp. 495–495, Jan. 2023, doi: https://doi.org/10.5455/jabet.2023.d144.

[25] M. Rasheed, M. N. Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "Solar PV Modelling and Parameter Extraction Using Iterative Algorithms," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012059, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012059.

[26] M. Rasheed, M. N. Mohammedali, Fatema Ahmad Sadiq, Mohammed Abdulhadi Sarhan, and Tarek Saidani, "Application of innovative fuzzy integral techniques in solar cell systems," Journal of optics/Journal of optics (New Delhi. Print), Jun. 2024, doi: https://doi.org/10.1007/s12596-024-01928-5.

[27] M. Rasheed, M. Nuhad Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "The numerical Calculations of Single-Diode Solar Cell Modeling Parameters," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012058, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012058.

[28] M. Rasheed, O. Alabdali, S. Shihab, A. Rashid, and T. Rashid, "On the Solution of Nonlinear Equation for Photovoltaic Cell Using New Iterative Algorithms," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012078, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012078.

Manuscript received on: 25/01/2023
Accepted on: 28/02/2023
Published on: 29/03/2023
https://doi.org/10.52688/ASP17593

[29] M. Rasheed, O. Y. Mohammed, S. Shihab, and A. Al-Adili, "Explicit Numerical Model of Solar Cells to Determine Current and Voltage," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012043, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012043.

[30] M. Rasheed, S. Shihab, O. Alabdali, A. Rashid, and T. Rashid, "Finding Roots of Nonlinear Equation for Optoelectronic Device," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012077, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012077.

[31] M. Rasheed, S. Shihab, O. Y. Mohammed, and A. Al-Adili, "Parameters Estimation of Photovoltaic Model Using Nonlinear Algorithms," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012058, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012058.

[32] M. Rasheed, SuhaShihab, O. Alabdali, and H. H. Hassan, "Parameters Extraction of a Single-Diode Model of Photovoltaic Cell Using False Position Iterative Method," Journal of Physics: Conference Series, vol. 1879, no. 3, p. 032113, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/3/032113.

[33] Manel Sellam, M. Rasheed, S. Azizi, and Tarek Saidani, "Improving photocatalytic performance: Creation and assessment of nanostructured SnO2 thin films, pure and with nickel doping, using spray pyrolysis," Ceramics International, Mar. 2024, doi: https://doi.org/10.1016/j.ceramint.2024.03.094.

[34] N. Assoudi et al., "Comparative examination of the physical parameters of the sol gel produced compounds La0.5Ag0.1Ca0.4MnO3 and La0.6Ca0.3Ag0.1MnO3," Optical and Quantum Electronics, vol. 54, no. 9, Jul. 2022, doi: https://doi.org/10.1007/s11082-022-03927-x.

[35] O. Alabdali, S. Shihab, M. Rasheed, and T. Rashid, "Orthogonal Boubaker-Turki polynomials algorithm for problems arising in engineering," 3RD INTERNATIONAL SCIENTIFIC CONFERENCE OF ALKAFEEL UNIVERSITY (ISCKU 2021), 2022, doi: https://doi.org/10.1063/5.0066860.

[36] S. M. H. AL-Jawad, M. Rasheed, I. M. Ibrahim, A. S. Sabber, and A. K. Elttayf, "Impact of Copper Doping on Nanocrystalline SnO2 Thin Films Synthesized by Sol-Gel Coating and Chemical Bath Deposition for Gas Sensor Applications," Journal of nano research, vol. 84, pp. 25–40, Sep. 2024, doi: https://doi.org/10.4028/p-4frfak.

[37] S. Shihab, M. Rasheed, O. Alabdali, and A. A. Abdulrahman, "A Novel Predictor-Corrector Hally Technique for Determining the Parameters for Nonlinear Solar Cell Equation," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022120, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022120.

[38] Selma, M. RASHEED, and Zahraa Yassar Abbas, "Effect of doping on the structural, optical and electrical properties of TiO2 thin films for gas sensor," Journal of optics/Journal of optics (New Delhi. Print), May 2024, doi: https://doi.org/10.1007/s12596-024-01913-y.

[39] T. Rashid, Musa Mohd Mokji, and M. Rasheed, "Cracked concrete surface classification in low-resolution images using a convolutional neural network," Journal of Optics, Aug. 2024, doi: https://doi.org/10.1007/s12596-024-02080-w.

[40] W. Saidi, Nasreddine Hfaidh, M. Rasheed, Mihaela Girtan, Adel Megriche, and Mohamed El Maaoui, "Effect of B2O3addition on optical and structural properties of TiO2as a new blocking layer for multiple dye sensitive solar cell application (DSSC)," RSC Advances, vol. 6, no. 73, pp. 68819–68826, Jan. 2016, doi: https://doi.org/10.1039/c6ra15060h.

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