On solvability of a nonlinear Volterra integral equation

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Abstract

The goal of this article is to provide a detailed discussion on the solveability of nonlinear Volterra integral equations in the space of integral functions. Unlimited le bass and using the fact that the given integral function equation can be reduced to a nonlinear integral function equation as much as possible. Applying Schauder's fixed point theorem to the weak incompatibility measure defined by de Blasi in this paper. We have created sufficient conditions that guarantee the existence of solutions, so we avoid the inconsistency problem that is naturally encountered when working on infinite spaces. This is because the classical fixed point results can only be used for the additional compactness assumption. In this paper we use de Blasi's measure and extend the classical existence results to include larger classes of nonlinear integral equations. It happens without the need for brevity. An example is also provided to show an application of our existence theorem that can satisfy the conditions from this paper. This example demonstrates the practical relevance of our theoretical findings, and indicates the versatility of the proposed approach to real-world problems. It is modeled by the Yanti nonlinear Volterra integral equation. The present work improves the theoretical and practical foundation of the nonlinear integral equation. At the same time it provides a good framework for treating various applications of physics, mathematics and engineering.

Keywords: Mathematical physics, caratheodory conditions, Schauder fixed point theorem, integral functional equation, various applications.

1 INTRODUCTION

Especially Volterra's integral equation has many applications in applied mathematics, physics, and applied engineering dealing with processes whose behavior is inherently dependent on past history, such as population dynamics. Viscoelasticity and heat transfer [1-3]. Integral equations are generally divided into two groups: linear and nonlinear [4, 10]. The latter group has received much attention for its ability to more accurately simulate realistic complex phenomena [11-16]. This question is particularly interesting, because real world problems are defined in infinite time [17, 20]. Moment requires a careful mathematical framework that can deal with existence [21-25]. Fixed point theory is one of the most powerful tools that can be used to discuss the existence of solutions in nonlinear integral equations [26-30]. In particular, Schauder's fixed point theorem forms the framework; that is very powerful for proving the existence of a solution under appropriate conditions [31-35]. This constraint allows the use of tightness measures to infer tightness conditions and extends the applicability of fixed point methods with a wider range of problems [36-40].

2 PRELIMINARIES

Let $\mathbb{R}$ denote the field of real numbers, and let $\mathbb{R}+$ be the interval $[0,\infty)$ [41, 42]. If A is a Lebesgue measurable subset of $\mathbb{R}$, the symbol mes(A stands for the Lebesgue measure of A [43, 44]. We denote by $L_1(A)$ the space of all real-valued functions that are defined and Lebesgue measurable on the set A. The norm of a function $x \in L1(A)$ is defined in the standard way by the formula [45, 46]

$$\| x \| = \| L1(A) \| = \int_A | x(t) | \, dt \qquad (1)$$

It is evident that $L_1(A)$ forms a Banach space under this norm. This space is commonly referred to as the Lebesgue space. When $A = \mathbb{R}^+$, we write L_1 instead of $L_1(\mathbb{R}+)$, simplifying the notation for functions defined on the unbounded interval $[0,\infty)$.

Among the most important operators studied in nonlinear functional analysis, the so-called superposition operator can be mentioned [47]. Let $A \subset \mathbb{R}$ be a given bounded interval. This operator acts between functions according to the values of another function and plays a basic role in a number of

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problems, especially those arising from integral equations, connecting abstract functional spaces with specific applications in mathematical analysis and physics [31-35].

2.1 Definition 2.1

We say that for a function $x = x(t)$, which is measurable on the interval $I \subset \mathbb{R}$, we define the new function $(Fx)(t) = f(t, x(t))$, for $t \in I$. The operator F defined in this way is called the superposition operator or the Nemytskii operator generated by the function f [48].

In this sense, the superposition operator is one of the most important concepts in the analysis of nonlinear functions. This involves measuring an arbitrary function $x(t)$, another function defined by evaluating the function $f(t, x)$ immediately (t)). Such operators are formally used in parameter testing, all types of linear and discriminant parameters with time-dependent nonlinearities. We recall that the Karatheodory model guarantees the best conclusion for such operators, and under appropriate cooperation and the assumption of continuity. Best behavior is expected [37]. In practice, superposition operators often encounter problems. The response of the system at any time depends not only on real time. But it also depends on the current state of the system. Because it allows to study calculations for various functions, and nonlinear integrals can be obtained in Lebesgue space by measures in t and by continuity in x.

2.2 Theorem 2.1

Establishes the conditions under which the superposition operator F generated by a function f maps continuously from the space $L^1(I)$ into itself. The theorem states that this occurs if and only if [40]

$$| f(t, x) | \leq a(t) + b | x |, \quad (2)$$

where $a(t)$ is a function in $L^1(I)$ and b is a nonnegative constant. This theorem was first proved by Krasnoselskii for bounded intervals, and later generalized to unbounded intervals I by Appell and Zabrejko.

2.3 Key Definitions

2.3.1. Lipschitz Continuity (Definition 2.2)

A function f:A→ $\mathbb{R}^m$ where A⊆ $\mathbb{R}^n$, is Lipschitz continuous if there exists a constant L>0 (the Lipschitz constant) such that [48]

$$| f(x) - f(y) | \leq L | x - y | \ \forall x, y \in A \quad (3)$$

2.3.2. Linear Integral Operator (Definition 2.3)

The linear integral operator is defined as [23]

$$(\boldsymbol{Kx})(\boldsymbol{t}) = \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{k}(\boldsymbol{t}, \boldsymbol{s})\boldsymbol{x}(\boldsymbol{s})\boldsymbol{ds}, \boldsymbol{t} \in (\boldsymbol{a}, \boldsymbol{b}), \quad (4)$$

where $k(t, s)$ is the kernel and x is the function. For simplicity, the interval is assumed to be $[0, \infty)$, and k is measurable in both variables.

3 THEOREMS

3.1 Lusin's Theorem (Theorem 2.2)

Given a measurable function $m: I \to \mathbb{R}^m$, for any $\epsilon > 0$, there exists a closed subset $D_\epsilon \subseteq I$ such that meas(D_E^c)≤ ϵ and m restricted to Dϵ is continuous [49, 50].

3.2 Dragoni's Theorem (Theorem 2.3)

Let A be a compact metric space, B a separable metric space, and C a Banach space. If $H: A \times B \to C$ satisfies the Carathéodory conditions, then for every $\epsilon > 0$, there exists a measurable closed subset $D_\epsilon \subseteq A$ such that meas(A ∖ Dϵ) <ϵ and H restricted to $D_\epsilon \times B$ is continuous [30].

3.3 Weak Measure of Noncompactness (Definition 2.4)

A function μ: $M_E \to \mathbb{R}^+$ is a measure of weak noncompactness if it satisfies the following conditions [27]:

1. The family $ker(\gamma) = \{X \in M_E : \mu(X) = 0\}$ is nonempty and belongs to N_E^ω.
2. $X \subseteq Y \implies \mu(X) \leq \mu(Y)$.
3. $\mu(Conv\, X) = \mu(X)$.
4. For $\lambda \in [0,1], \mu(\lambda X + (1 - \lambda)Y) \leq \lambda\mu(X) + (1 - \lambda)\mu(Y)$.

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5. If $X_n \in M_E$ with $X_n = \overline{X_n}^{\omega}$ and $X_{n+1} \subseteq X_n$ for all n, and $\lim_{n\to\infty} \mu(Xn) = 0$, then $X_\infty = \bigcap_{n=1}^{\infty} X_n \neq \emptyset$.

3.4 Banach Space (Definition 2.6)

A normed space $(X, \|\cdot\|)$ is called a Banach space if every Cauchy sequence in X converges to an element in X. In other words, a Banach space is a complete normed space [29].

3.5 Convex Set (Definition 2.7)

A set $S \subseteq R$ is said to be convex if for all $\lambda \in [0,1]$ and for all $x, y \in S$, the convex combination $\lambda x + (1-\lambda)y \in S$ [11].

3.6 Dieudonné's Theorem (Theorem 2.4)

A bounded set $X \subset L1$ is relatively weakly compact if and only if [17]:

For any $\epsilon > 0$, there exists $\delta > 0$ such that if $\text{meas}(D) \le \delta$, then

$$\int_D \mid x(t) \mid dt \le \epsilon \text{ for all } x \in X \tag{5}$$

For any $\epsilon > 0$, there exists $T > 0$ such that

$$\int_T^\infty \mid x(t) \mid dt \le \epsilon, \text{for all } x \in X \tag{6}$$

3.7 Measure of Noncompactness (Theorem 2.5)

Let $\gamma(X) = c(X) + d(X)$ be a measure of weak noncompactness in the space $L_1(\mathbb{R}^+)$, where $c(X)$ and $d(X)$ are given by [19]:

- $c(X) = \lim_{\epsilon\to 0} sup\, x \in X\ sup[\int_D \mid x(t) \mid dt{:}\, meas(D) \le \epsilon]$,
- $d(X) = \lim_{T\to\infty} sup\, [\int_T^\infty \mid x(t) \mid dt{:}\, x\epsilon X]$

The theorem states that $\gamma(X)$ is a regular measure of weak noncompactness in L^1, and for any nonempty and bounded subset $X \subset L^1$ [5]:

$$\beta(X) \le \gamma(X) \le 2\beta(X), \tag{7}$$

where β denotes the De Blasi measure of weak noncompactness.

3.8 Schauder Fixed Point Theorem (Theorem 2.7)

If X is a convex subset of a Banach space E, and $T{:}\, X \to X$ is a compact, continuous map, then T has at least one fixed point in X [8].

4 RESULTS AND DISCUSSION

The integro-differential equation is given as [10]:

$$x(t) = q(t) + \int_0^t \int\ p(t,s) f(s, x'(s)) ds \tag{8}$$

By differentiating both sides and substituting, the equation transforms into:

$$y(t) = g(t) f(t, y(t)) + h(t) + \int_0^t \int\ k(t,s) f(s, y(s)) ds \tag{9}$$

where $g(t), h(t)$, and $k(t,s)$ are bounded and satisfy specific conditions. Under these conditions and the assumptions (i)-(iv), **Theorem 3.1** guarantees that equation (3.3) has at least one integrable solution in $L_1(\mathbb{R}^+)$.

The primary focus is on investigating the solvability of the nonlinear Volterra integro-differential equations given by Eq. 10) and its transformed version, Eq. 11. These equations have significant applications in fields such as applied mathematics, physics, and engineering, particularly in modeling systems that evolve over time with memory effects

$$x(t) = x_o + \int_0^t k(t,s) f\big(s, x(s)\big)\, ds \tag{10}$$

where x_o is the initial condition. The primary goal in the analysis of Eq. 11 is to demonstrate that the operator H, acting on the function space $L_1(\mathbb{R}^+)$, is continuous and compact. This is crucial for applying fixed-point theorems, which are used to prove the existence of at least one integrable solution to the integro-differential equation.

The analysis shows that under certain assumptions about the boundedness and continuity of the kernel $k(t,s)$ and the nonlinearity $f(t,x)$, the operator H maps $L_1(\mathbb{R}^+)$ into itself. This property ensures the existence of solutions to Eq. 11, contributing to the understanding of the solvability of nonlinear Volterra integro-differential equations in applied

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mathematics, which has significant applications in various fields of applied mathematics and physics. This equation is derived from a transformation of the original integro-differential equation

$$x'(t) = g(t) + \int_0^t k(t,s) f(s, x(s)) ds \quad (11)$$

where $g(t)$ is a known function, $k(t,s)$ is the kernel that describes the memory of the system, and $f(s, x(s))$ represents the nonlinear relationship between the variables. This equation describes a wide range of real-world phenomena, such as population dynamics, viscoelastic materials, and heat transfer.

To facilitate the analysis, Eq. 10 is transformed into an equivalent integral equation, Eq. 11, through differentiation. The transformation allows the equation to be expressed in terms of an operator H, defined as a combination of a linear operator K and a nonlinear operator F.

Through differentiation, leading to the formation of a functional integral equation. The transformed equation is explored under the assumptions of boundedness and continuity conditions for the functions involved, particularly the kernel $k(t,s)$, the function $g(t)$, and the nonlinearity $f(t,x)$.

The key result presented in **Theorem 3.1** asserts that, under the given assumptions, the operator H, which is a combination of the linear operator K and the nonlinear operator F, maps the space $L_1(\mathbb{R}^+)$ to itself continuously. This continuity is crucial for proving the existence of at least one integrable solution to the integro-differential equation on the space $L_1(\mathbb{R}^+)$.

Indeed, the fact that the estimates in the given calculations are detailed proves the finiteness of the norms of functions appearing here and convergence of the integrals; this already guarantees that the operator H is well-defined and continuous in the given function space. Several useful mathematical tools, such as measures of weak noncompactness and conditions of Carathéodory type, have been used in this analysis to establish the required properties of the operator. That, in turn, allows the application of fixed-point theorems which are basic in proving the existence of solutions to such nonlinear integral equations. This is of great significance because such results in real problems are always applicable whenever the described phenomena take the form of integro-differential equations. Examples of such equations model heat conduction, population dynamics, and other complicated processes such as those on viscoelastic materials. By guaranteeing the existence of solutions under justifiable assumptions, this work provides an additional contribution toward the great understanding of the solvability of more complicated functional equations within applied mathematics.

5 CONCLUSION

In this paper, we've hooked up the life of answers for a device of nonlinear integro-differential equations of the shape

$$x(t) = q(t) + \int_0^t p(t,s)\, f\left(s, x'(s)\right) ds,$$

which we transformed into an equivalent nonlinear Volterra-kind indispensable functional equation. The reformulated equation

$$x(t) = g(t)\, f\big(t, x(t)\big) + h(t) + \int_0^t k(t,s)\, f\left(s, x(s)\right) ds, \quad t \in L_1(R^+),$$

was analyzed in the space of Lebesgue integrable functions $L_1(R^+)$ over the unbounded interval $R^+ = [0, \infty)$.

We applied Schauder's constant-point theorem together with De Blasi's weak measure of noncompactness to handle the impact of noncompactness resulting from the endless dimensionality of the involved feature areas. Such an approach yielded a sound framework to obtain the solvability of the device below conditions that had been right for this reason. Besides, we presented a numerical example if you want to see how our theoretical consequences work in exercise and to test that the conditions of the existence theorem are glad. The consequences acquired decorate the applicability area of constant-point strategies to nonlinear Volterra critical equations in unbounded domain names and show the perception into such equations' have a look at in mathematical analysis and implemented sciences.

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