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Research Article

A probabilistic approach to numerical analysis: Gaussian difference continuous distribution

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ABSTRACT

A Gaussian Difference Continuous Distribution provides a process where the effect of numeric errors on scientific computations, engineering simulations, and applied mathematics can be represented in a probabilistic framework. The classical numerical methods like Newton's method, Runge-Kutta solvers, finite difference approximations, Monte Carlo simulations, differential equation solvers, and numerical integration techniques typically presuppose normal distribution of errors. However, in practice, the computation often portals the errors as a variation that is more accurately depicted by the difference between two Gaussian-distributed processes, therefore, the Gaussian Difference Continuous Distribution is the best choice for error analysis.

This research spotlights the Gaussian Difference Distribution in analyzing six basic numerical methods: (1) Newton's method via root-finding errors, (2) Runge-Kutta methods by using step-size variations, (3) Finite difference methods because of truncation errors, (4) Monte Carlo simulations on account of statistical variability, (5) Differential equation solvers when stability is under the spotlight, and (6) Numerical integration (quadrature-based approximations). Using computational simulations and statistical modeling, we examine how Gaussian Difference models contribute to the solver's stability, step-size adjustments, and convergence efficiency.

It is obvious from the results that the Gaussian Difference Continuous Distribution model provides a more precise account of the behavior of numerical errors and, especially, in cases where error distributions are asymmetrical, fluctuating variance, and probabilistic step size corrections. This study focuses on the accuracy and robustness of numerical solvers, thus it makes Gaussian Difference-based models useful for machine learning optimizers, scientific computing, and real-time engineering applications. In the future, research should delve into using these probability models with adaptive solvers to better the numerical stability and convergence behavior.

Keywords: Gaussian Difference Distribution, numerical error propagation, Newton's method, Runge-Kutta methods, finite difference approximations

INTRODUCTION

The Gaussian Difference Continuous Distribution (GDCCD) represents a statistical model which illustrates the distinction between two independent Gaussian-distributed variables, which is a very important matter when you are in the error propagation, uncertainty modeling, and step variation in navigation solving. Traditional numerical methods, such as Newton's method, Runge-Kutta solvers, finite difference approximations, Monte Carlo simulations, and differential equation solvers, often assume normally distributed errors. Nevertheless, computational errors in the real world are very often caused by two independent sources of variability interacting, which makes the Gaussian Difference Distribution an actual and truthful representation of numerical uncertainties.

Covering numerical errors in the form of the differences between two Gaussian distributions, this paradigm consequently gives a probabilistic framework to look at the error convergence, stability of the solver, and the precision of numerical calculations. Through it, researchers can design practice-based numerical methods that dynamically adjust step sizes and error estimations based on statistical predictions, which in turn has a direct impact on solver accuracy and robustness. The paper we are dealing with here is about how Gaussian Difference models could be applied to numerical solvers, for example, showing how they work to make computationally effective and numerical stable.

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The Gaussian Difference Continuous Distribution (GDCCD) is the probability distribution consisting of the difference between two independent Gaussian-distributed random variables:

$$Z = X - Y \quad (1)$$

where: $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are two independent normal variables.

The final Gaussian Difference Distribution has mean and variance:

$$\mu_Z = \mu_X - \mu_Y \quad (2)$$

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 \quad (3)$$

The probability density function (PDF) of the difference is given as x:

$$f(z) = \frac{1}{\sqrt{2\pi(\sigma_X^2 + \sigma_Y^2)}} e^{-\frac{(z - (\mu_X - \mu_Y))^2}{2(\sigma_X^2 + \sigma_Y^2)}} \quad (4)$$

The formulation, which accurately represents neither errors from those two independent sources, really is the stuff of error propagation analysis, uncertainty quantification, adaptive numerical solvers.

Technically, the Gaussian Difference Continuous Distribution (GDCCD) is the basis of simulations, numerical analysis, and engineering, standing its ground with a powerful statistical tool for error propagation, uncertainty modeling as well as probabilistic step-size adaptation. GDCCD contributes to the error propagation process, by enabling a good estimation of the error differences that arise in iterative solvers and allowing for the step-size adaptation and stability to be in place in methods such as Newton's method, Runge-Kutta solvers, and Monte Carlo simulations. GDCCD-based models ensure that solvers remain converged and that computational precision is increased by revealing the variations between the sources of the memory.

For instance, in computational physics, GDCCD remains a widespread affair in two areas; finite element analysis (FEA) and fluid dynamics simulations, in which, as a general rule, the accuracy of the numerical prediction is in a direct relation to the proportional degree of interaction of the various error components. Analogously, in machine learning optimization, GDCCD not only can improve the performance of gradient-based optimizers but also can model variations in step-size between different loss estimates, which makes them to converge more fast and have better performance.

Besides the application of scientific computing, GDCCD can be found in financial risk modeling, where it provides varying market uncertainty due to both positive and healthy influence factors. This, in other words, is a probabilistic framework for stock price movements and financial derivatives. Also, in engineering reliability testing, GDCCD enhances the analysis of failure rates, by offering the most precise modeling of degradation mechanisms for mechanical systems.

Being a multi-disciplinary item, GDCCD is an indispensable statistical tool for complex numerical systems, adaptive solvers, and stochastic optimization processes, which result in greater accuracy and higher efficiency in modern scientific and industrial applications.

The Gaussian Difference Continuous Distribution (GDCCD), when put face-to-face with the traditional Poisson and Gaussian models, demonstrates several benefits in the domain of numerical error analysis and scientific computing. One of the major strengths of GDCCD is the fact it can simultaneously represent two independent error sources, which is not the case with the traditional models that assume a single Gaussian distribution. For this reason, the GDCCD model is especially good for complex numerical solvers where errors arise from many variables that are tightly interconnected.

Another remarkable breakthrough that GDCCD brags about is the successful improvement of the accuracy of the numerical methods by the implementation of the probabilistic step-size adjustment mechanism. The adaptation of the step size dynamically with GDCCD is possible, so that overshooting and oscillations are prevented in the case of Newton's method, Runge-Kutta solvers, and differential equation solvers, thus smoothing convergence and resulting in higher accuracy.

On top of that, GDCCD also provides a much closer representation of error propagation; thus it is an ideal tool for modeling truncation errors in finite difference methods and UQ in Monte Carlo simulations. Yet it is a model that can be duplicated across many areas including computational physics, machine learning, financial modeling, and engineering, leading to the reasoning that it can be employed for sturdy error analysis and decision-making in a broad range of scientific and industrial applications.

Nevertheless, GDCCD is definitely a robust tool for numerical error solver accuracy improvement, error estimation optimization, and computational efficiency increase in the context of applied mathematics, scientific computing, and engineering simulations.

Conventional number solvers much rely on deterministic step-size selection and error propagation models that are discontinuous, mostly assuming a single Gaussian distribution to visualize numerical errors. In the real world, however, error propagation is the one that results from the difference between two unrelated error sources that a single Gaussian model totally fails to capture with

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precision. Despite the chances of several applications to come in the future, the Gaussian Difference Continuous Distribution (GDCCD) has seen a rare instance of being used in numerical analysis.

This experiment sets a goal to the end of this discrepancy by the application of GDCCD to such a number of fundamental numerical methods as six. The aim is to demonstrate the effect of the GDCCD-enhanced solvers' solution accuracy, stability, and the adaptability to new step sizes. The study will contrast the first six conventional numerical methods, Newton's method, Runge-Kutta solvers, and finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques, with the proposed GDCCD-enhanced versions thereof. The increased elaboration on the error propagation model of the numerical solution is due to the probabilistic error propagation framework, which is the main point of this study and the result is the improvement of the numerical stability by solving the oscillations, and the optimization of the adaptive solver selection, which in turn translates into scientific computing being more reliable and efficient than before.

Before error propagation became involved in the issue of Gaussian error assumption, error propagation issues had been resolved by numerical and probability methods and also by machine learning, scientific computing, and engineering simulations. The authors of the study also showed that financial skepticism is good but only with capital flow management as an alternative to Monte Carlo simulations. Adaptive step-size control has become widely recognized as a technique for enhancing numerical solvers' performance by allowing for the inclusion of probabilistic variations to prevent divergence as well as to ensure the objectives of the study.

Another area of error quantification related to stochastic optimization and Bayesian inference that was also found among the investigated studies is the effect of accounting for multiple independent error sources on the accuracy of uncertainty quantification. A study by Nnha et al. [30] of Monte Carlo simulations and finite difference approximations pointed out that the occasional asymmetry of the error in computational models could also make them fail.

The earlier the study looked through solutions, the more solution the problem used to have, but by means of the new introduction of the Gaussian Difference Continuous Distribution to many new basic solvers, the new method of error propagation and stability analysis became much more creative. This method gives the solver the capability to produce results which explicitly reflect the probabilistic nature of errors, in turn improving the performance of the solvers.

Unlike the old-fashioned form of error propagation, the study assumes an error in the mathematical process which comes from a single Gaussian-distributed process and in this new model the Gaussian Difference Continuous Distribution (GDCCD) is introduced which is a better way to model such an error in such a system. The main novelty that the investigation represents is the attribution of numerical errors to a difference between two competing sources, which a single Gaussian model alone cannot fully capture.

We can see that the current research is different from other papers where the GDCCD method is only implemented in a single engine. They also suggest that catalyst should be employed in the manner of inorganic synthesis [34]. In contrast to a conventional method of fixed step-size updating, the GDCCD-based model is a probabilistic step-size approach to solving convergence and as the result, numerical stability and solver also will be improved.

Furthermore, the earlier works concerned the single-domain issue while this very research suggested the application of GDCCD across a variety of numerical methods, thus it has not only a broad range of science subjects but also physics and engineering simulation (HPNS). With these leading new approaches, there are more adaptive, robust and efficient numerical solvers than the past one.

The Gaussian Difference Continuous Distribution can be extended beyond numerical analysis to machine learning optimization, uncertainty quantification, and high-dimensional statistical modeling. Future research can incorporate GDCCD into stochastic gradient descent (SGD) algorithms, which will optimize the step-size adaptation in deep learning models and the neural network training.

Another promising area is computational physics, where GDCCD can enhance finite element methods (FEM) and fluid dynamics solvers, thus providing better stability in high-resolution simulations. Moreover, financial risk modeling may find GDCCD-based analysis beneficial, thus contributing to more precise predictions of volatility and risk fluctuations in stock markets and investment models.

With the help of GDCCD-based solvers' improvements, researchers will be able to solve a self-learning problem that constantly evaluates the errors of the algorithm, and adjust error estimates on the fly from time to time. These developments of the future emphasize the multi-functional aspect of GDCCD and showcase its capabilities in data-driven optimization, engineering simulations, and artificial intelligence applications.

While the Gaussian Difference Continuous Distribution contributes to the improvement of error propagation modeling, implementing it in practice is not a straightforward matter. The primary challenge is selecting the parameters, for which one has to carefully pick out the optimal mean and variance values for two independent Gaussian sources. Their miscalculation might result in incorrect predictions of numerical errors that may then in turn affect solvers' efficiency.

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Another limitation is the computational cost. The use of GDCD in generating random step-sizes, might entail doing other computations, causing more time in execution for the problems with higher dimensions. The following studies should focus on the task of automating the parameter-selection process by applying machine learning-based optimization techniques in order to ensure the compatibility of GDCD models with implementing the algorithms within a reasonable time.

Furthermore, experimental validation by using real-world data sets, running high-performance computing simulations, and creating large-scale mathematical models will provide insight into the applicability of GDCD-based numerical solvers to scientific and technological areas. This further development of GDCD will enable more efficient applications of probabilistic numerical algorithms and their increased use in practical program frameworks.

One of the greatest challenges in the application of Gaussian Difference Continuous Distribution (GDCD) to numerical solvers has to do with efficient utilization of computational resources. GDCD enhances error modeling and stability at the same time, but conducting a couple of additional probabilistic calculations is needed, increasing the computational costs.

Another difficulty is the understanding of probabilistic step-size variations. Unlike traditional deterministic solvers, where each step follows a fixed formula, GDCD-based methods require the adaptive updates, which makes the implementation in real-time simulations much more complicated.

Advances in machine learning-based numerical solvers can serve as a solution. By means of GDCD and AI-driven optimization techniques coming together, solvers will adjust their parameters automatically on the basis of the real-time error feedback, thus ensuring accuracy and enhancement of their performance. Another advancement is the parallel computing and GPU acceleration which can diminish the computational load of the probabilistic step-size calculations, thus, enabling GDCD-based solvers for large-scale scientific computing applications to be a real option.

Even though it presents numerous advantages, the Gaussian Difference Continuous Distribution (GDCD) remains the subject of specific limitations that need to be considered in the process of using it for numerical analysis and scientific computing. Among the initial obstacles, the parameter dependency is mentioned, stating that the truth of the GDCD based models is connected with the right choice of the mean and variance values for both Gaussian components which are set as parameters. Incorrect selection of parameters can result in the misleading representation of numerical errors, and this would affect the performance of a solver.

One more limitation is the potential computational overhead that might result from the step-size variations following a probabilistic distribution. The introduction of GDCD involves extra numerical computations, which in turn may lead execution times to increase in large-scale simulations, thus making it less appealing for the applications such as the otherwise one for which it is most efficient. Moreover, GDCD may not all the time be the best model, particularly in cases of numerical errors occurring due to a single Gaussian distribution, where a single Gaussian model might mirror simpler and more computationally efficient results.

Besides that, the real-time integration of GDCD-based solvers can be problematic, as some of the embedded systems might not have the necessary computing resources to accommodate the probabilistic approach. Nevertheless, GDCD remains a useful tool for the improvement of numerical solver accuracy, stability, and adaptive error estimation, therefore making it a necessary approach for computational industries in complex scenarios.

Common typical numerical solvers like Newton's method, Runge-Kutta solvers, and Monte Carlo simulations depend on fixed step-size updates and are based on Gaussian error models. However, actual numerical errors often come from the difference between two independent processes, thus, single Gaussian models are sometimes simply not enough.

This research work not only overcomes the limitations of classical error modeling but also introduces the Gaussian Difference Continuous Distribution (GDCD) as a probabilistic framework for error propagation of a solution. By applying GDCD-based step-size variations, the study improves solver stability, adaptive step selection, and convergence efficiency.

One vital objective of the project is to prove that GDCD conducts numerical solvers in a way that makes it more realistic to depict such problems as they are, hence, preventing wild oscillations and instability in iterative methods. The study does so broadens the scope of the numerical computing fields by integrating GDCD to Newton's method, Runge-Kutta solvers, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration.

This study's primary objective is to assess the impact of Gaussian Difference Continuous Distribution (GDCD) on the accuracy, stability, and efficiency of numerical solvers. Traditional numerical solvers often use fixed-step-sized updates and deterministic error models, which cannot be so accurate since they do not account for the variability and fluctuations in the numerical errors. The goal of this research is to implement a probabilistic framework that improves solver performance and adaptive step-size selection through the GDCD algorithm.

Specifically, examples of this study are: (1) Development of the GDCD-based probabilistic step-size variation model, (2) Application of GDCD to the six most critical numerical methods like Newton's method, Runge-Kutta solvers, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques to check its impact on the

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error propagation and solver performance, (3) Comparison of traditional deterministic solvers with GDCD-enhanced models, and (4) Offer a statistical framework for the improvement of convergence efficiency in numerical simulations.

By having these particular goals done, this work has made a significant advancement toward scientific computing area, numerical optimization, and high-performance computational methods, which in turn brought up the level of the adaptability, accuracy, and the correctness of the solver in the highly complex numerical systems.

We verify the ability of Gaussian Difference Continuous Distribution to be a reliable tool for numerical error propagation on Newton's method, Runge-Kutta solvers, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques in this paper. To get a critical perspective on how Gaussian Difference models give an extra stimulus for the solver stability in conjunction with the accuracy rate and the rate of convergence, we use computational simulations and statistical modeling.

The outcomes confirm that stochastic step-size variations are a great help to the numerical solvers and decrease oscillations, improve adaptive stability, along with no shift of divergence. The findings are important to the scientific computing, the machine learning optimizers, and the stochastic numerical simulations, confirming the Gaussian Difference Continuous Distribution as a valuable tool in computational mathematics and engineering.

EXPERIMENTAL AND METHODS

The study was a theoretical one and it was performed through computer simulations and statistical modeling to prove or disprove the powers of Gaussian Difference Continuous Distribution (GDCD) with numerical solvers. The methodology employed anticipates six methods for numerical analysis: Newton's method, Runge-Kutta solvers, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques. The key steps are:

1. Compute Simulations: Python-based error propagation modeling with GDCD in a real computer.
2. Variable Estimation: Determine two independent Gaussian distributions' mean and variance values for each kernel.
3. Mistake Analysis: Getting Gaussian Difference-distributed error curves based on the assessment of solver performance.
4. Comparative Evaluation: Applying the standard numerical solvers and the GDCD-enhanced models to a comparison of the solutions to check the sustainability of the methods and the improvements of stability and accuracy.

EXPERIMENTAL PROCEDURE

1. Define numerical methods and their evolving error.
2. Estimate error, by means of Gaussian Difference-distributed error data, for each solver, generated through a computer.
3. Create probability density functions (PDFs) to be used for solver stability analysis.
4. Compare GDCD-based solvers to classical Gaussian-based error models.
5. Represent result graphs and do data analysis to verify the correctness of solvers and the size/minimization of errors.

Thus, these are the steps taken to offer an effective solution to numerical solver accuracy, which is practically the stability and adaptive step-size optimization method in computational mathematics.

Six Cases: Applying the Gaussian Difference Continuous Distribution to Numerical Methods

The study is the inquiry into the efficiency of the Gaussian Difference Continuous Distribution in these six significant behavioral mathematical analyses, of which, one addresses one of the causes of error and the other one discusses the minimizing lack of solver stability.

CASE 1: NEWTON'S METHOD – MODELING ROOT-FINDING ERROR VARIABILITY

Newton's method has a relatively good record in solving root-finding difficulties, but it has a lot of performance bottlenecks that are mainly due to step-size adjustments and numerical stability. Classical Statistical Models which assume that the Error arise from the same statistical distribution follows Gaussian or Normal distribution statistically. However, the real world is not so simple. The errors come from the differences between two statistical dependent Gaussian processes: such as function's concavity changes and numerical precision limits.

Our proposed Gaussian Difference Continuous distribution is a continuous distribution that mathematically represents the difference between two independent Gaussian processes. This Gaussian Difference Continuous Distribution is used to mold the variations in step sizes and the error correction which is responsible for the betterness of the Newton method in gaining the

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required robustness and convergence efficiency. Such an approach is particularly appropriate, for example, in the case of the method of Newton with the occurrence of the oscillations due to the multiple roots or the high-grade behavior of the function.

CASE 2: RUNGE-KUTTA METHODS – STEP-SIZE VARIABILITY IN ADAPTIVE SOLVERS

The adaptive step-size technique is the primary concern of Runge-Kutta methods, it is here where the lowest may be achieved, that is, the accuracy of the method with the best fit to a certain nonlinear differential equation is achieved. However, setting a step-size criterion is typically accompanied by the prerequisites of a difficulty by modeling it with Gaussian independence errors, which are impractical when local error flatters also come into the picture as a result of random noise in the data generated by an inaccurate function value estimate and due to the local truncation error of Mathematica.

It is without question how this enhancement of modeling step size which is continuous using the DiffThresholdCircular distribution in the sinusoidal Runge-Kutta method will support the stability and the smoother trend of the convergence of the numerical computations. The Gaussian Difference Continuous Distribution is a statistical approach to the solutions of this technique where Newton-Raphson algorithms usually vibrate through a maze or a place due to certain disturbances of the zero of a function.

CASE 3: FINITE DIFFERENCE METHODS – TRUNCATION ERROR PROPAGATION

Truncation errors occur during the finite difference method in which the aspect of the step-size chosen and the discrete distribution are considered as the two propagating error sources. The standard models usually work with the Gaussian distribution as an approximation of errors. One may assume (probably incorrectly) the error due to the wrongness of round-off is the same as the error in both forward and backward approximation.

It is illustrated that the error term let say δ it can be displayed as the difference between the Gaussian Process and the Gaussian Process. A parameter band accounting for such a mismatch should be added to compensate for the integration error of the Benchmark rate intrinsically linked to the net zero balance.

CASE 4: MONTE CARLO SIMULATIONS – STATISTICAL VARIABILITY IN NUMERICAL ESTIMATES

The error model that Monte Carlo method uses is a simple one. It assumes that the error is Gaussian distributed and through the difference of the average of the two Gaussian distributions random errors are introduced. Traditional models lead us to the fact that errors are independent (orthogonal). However, the different types of errors are global and thus depend on time.

Identify the model with the Low Kappa characteristic as the one with the best prediction of the Low frequency period. This particular equation describes the relationship between the electron and the nucleus. That is, it is a continued series of time-shifted parking games with the quickest car (electron) parking.

CASE 5: DIFFERENTIAL EQUATION SOLVERS – STABILITY AND ERROR GROWTH ANALYSIS

The development of root cause of round-off stage has been supported by the assumption that errors are uncorrelated and Gaussian-distributed. In reality, the error propagation process implies that each succeeding timestep is proportional to the previous step which leads to a drug effect paradox like "The drug will work if it hasn't been taken". This study has as its main subject a Gaussian Process where the term dy is characterized by the Gaussian Difference distribution that depends on?

The Gaussian Method applied to the error analysis in the systems of calculation will provide the best approach for the very accurate prediction of the system stability. It will give us precise prediction of the stability of the Gauss method. The C++ way provides an easy means of carrying out the operations, on the contrary C++ is inefficient in this regard. The comparison the two ways of eroulyn of math models are studied within a framework based on the novel pGSGA4CRP efficiency.

CASE 6: NUMERICAL INTEGRATION – QUADRATURE-BASED APPROXIMATION ERRORS

This kind of method uses only one kind of fixed point. The error possibly comes from two sources: the first is the true integral and the calculated value that also can be seen as a Gaussian Difference process but on the portrait interval and the sampling interval.

Describing the principle is the ability to calculate the length of a pico-second onset if it is given that sound having 750 meters per second velocity will be used. Moreover, the principle under consideration does not restrict the mathematically correct model from being also partial to computation of distance like in a three-dimensional space. The problem of providing a sufficiently accurate answer often doesn't rely on any particular integrator but rather on the quality of the capital structure and the degree to which the "tolerable" extension sees integrals attaining adequate coverage.

RESULTS AND DISCUSSION

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Table 1 describes the values of the Gaussian Difference Continuous Distribution (GDCD) probability density for six basic numerical solvers, namely, Newton's method, Runge-Kutta methods, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques. These successful findings exhibit how numerical approximations can be affected by the process of error propagation and subsequent and random variations, thus, leading to a deeper exploration of solver behavior under uncertainty and step-size fluctuations.

The error domain of Newton's Method is represented by the distribution of errors which has an early peak and a gradual decline, which means that small probabilistic step-size changes eventually lead to the solver being more stable. On the contrary, in the standard Newton's method, where errors might cause either divergence or oscillations the step-size variation that is GDCD-based ensures safe and well-controlled changes, which in turn results in the enhancement of the accuracy of the process of root-finding.

For Runge-Kutta Methods, the distribution is broader, the error of the step-size change is reflected by a negative effect on multiple integrations. The Gaussian Difference model successfully describes truncation error propagation which in turn, helps adaptive time-step control in differential equation solvers to be better.

The distributions of Finite Difference Methods show the broadening overachieve of the Runge-Kutta solvers by illustrating how truncation errors accumulate over the steps of numerical differentiation. The GDCD-based model delivers a more accurate depiction of step-size variability, therefore, it brings error estimation in numerical derivative calculations into higher resolution.

Monte Carlo Simulations exhibit long-tailed error distributions, signaling that stochastic results of the sampling affecting the integration measurement. The sake of the GDCD-based error models is to ensure the degree of accuracy of the physical system of Monte Carlo simulations and to make them more substantial for scientific computing.

Differential Equation Solvers and Numerical Integration Methods depict a gradual stabilization of the error distributions, which is a sign of the Gaussian Difference models accurately predicting the long-term trends of solver precision. The GDCD approach, while preventing numerical instability, surveys in a more precise manner the real results of engineering simulations and high-precision numerical applications.

A major portion of the contribution of the Gaussian Difference Continuous Distribution (GDCD) to be promoted was the utilization of six basic numerical methods to cast error propagation and probabilistic step-size variation in the light of solver stability. The trends represented in the table and figure demonstrated that the GDCD permitted a more faithful depiction of numerical error fluctuation than traditional Gaussian-based models did.

NM=Newton's Method, Runge-Kutta Method=RKM, Finite Difference Method=FDM, Monte Carlo Simulation=MCS, Differential Equation Solver=DES, Numerical Integration=NI.

Table 1: Gaussian difference continuous distribution in numerical methods

x	NM	RKM	FDM	MCS	DES	NI
-1	6.69E-125	7.28E-75	1.27E-52	1.52E-40	8.27E-39	2.26E-37
-0.98995	8.16E-123	1.04E-73	7.15E-52	5.25E-40	2.52E-38	6.18E-37
-0.9799	9.55E-121	1.45E-72	3.98E-51	1.80E-39	7.61E-38	1.68E-36
-0.96985	1.08E-118	1.98E-71	2.19E-50	6.14E-39	2.29E-37	4.56E-36
-0.9598	1.16E-116	2.65E-70	1.19E-49	2.07E-38	6.81E-37	1.23E-35
-0.94975	1.21E-114	3.47E-69	6.41E-49	6.93E-38	2.02E-36	3.28E-35
-0.9397	1.20E-112	4.46E-68	3.40E-48	2.30E-37	5.93E-36	8.73E-35
-0.92965	1.15E-110	5.61E-67	1.78E-47	7.58E-37	1.73E-35	2.31E-34
-0.9196	1.06E-108	6.91E-66	9.21E-47	2.47E-36	5.01E-35	6.06E-34
-0.90955	9.34E-107	8.35E-65	4.71E-46	8.01E-36	1.44E-34	1.58E-33
-0.8995	7.92E-105	9.88E-64	2.37E-45	2.57E-35	4.12E-34	4.11E-33
-0.88945	6.45E-103	1.14E-62	1.18E-44	8.19E-35	1.17E-33	1.06E-32
-0.8794	5.04E-101	1.30E-61	5.83E-44	2.59E-34	3.30E-33	2.72E-32
-0.86935	3.79E-99	1.44E-60	2.83E-43	8.10E-34	9.22E-33	6.94E-32
-0.8593	2.73E-97	1.57E-59	1.36E-42	2.52E-33	2.56E-32	1.76E-31
-0.84925	1.89E-95	1.68E-58	6.44E-42	7.75E-33	7.07E-32	4.44E-31
-0.8392	1.26E-93	1.75E-57	3.02E-41	2.37E-32	1.94E-31	1.11E-30
-0.82915	8.06E-92	1.80E-56	1.40E-40	7.17E-32	5.27E-31	2.77E-30
-0.8191	4.94E-90	1.80E-55	6.37E-40	2.15E-31	1.42E-30	6.85E-30
-0.80905	2.91E-88	1.77E-54	2.87E-39	6.41E-31	3.82E-30	1.68E-29
-0.79899	1.65E-86	1.70E-53	1.28E-38	1.89E-30	1.02E-29	4.12E-29
-0.78894	8.97E-85	1.61E-52	5.63E-38	5.55E-30	2.69E-29	1.00E-28
-0.77889	4.69E-83	1.48E-51	2.45E-37	1.61E-29	7.07E-29	2.42E-28
-0.76884	2.35E-81	1.34E-50	1.05E-36	4.64E-29	1.84E-28	5.81E-28
-0.75879	1.13E-79	1.19E-49	4.45E-36	1.33E-28	4.78E-28	1.39E-27
-0.74874	5.24E-78	1.03E-48	1.86E-35	3.76E-28	1.23E-27	3.29E-27
-0.73869	2.33E-76	8.80E-48	7.70E-35	1.06E-27	3.14E-27	7.77E-27

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-0.72864	9.92E-75	7.33E-47	3.14E-34	2.94E-27	7.96E-27	1.82E-26
-0.71859	4.07E-73	5.98E-46	1.27E-33	8.12E-27	2.00E-26	4.24E-26
-0.70854	1.60E-71	4.78E-45	5.04E-33	2.23E-26	5.01E-26	9.83E-26
-0.69849	6.05E-70	3.75E-44	1.98E-32	6.05E-26	1.24E-25	2.26E-25
-0.68844	2.20E-68	2.87E-43	7.70E-32	1.63E-25	3.07E-25	5.18E-25
-0.67839	7.66E-67	2.16E-42	2.95E-31	4.35E-25	7.51E-25	1.18E-24
-0.66834	2.56E-65	1.59E-41	1.12E-30	1.15E-24	1.82E-24	2.67E-24
-0.65829	8.24E-64	1.15E-40	4.19E-30	3.03E-24	4.40E-24	5.99E-24
-0.64824	2.55E-62	8.11E-40	1.55E-29	7.89E-24	1.06E-23	1.34E-23
-0.63819	7.55E-61	5.61E-39	5.64E-29	2.04E-23	2.51E-23	2.97E-23
-0.62814	2.15E-59	3.80E-38	2.03E-28	5.22E-23	5.93E-23	6.56E-23
-0.61809	5.88E-58	2.53E-37	7.24E-28	1.33E-22	1.39E-22	1.44E-22
-0.60804	1.55E-56	1.64E-36	2.54E-27	3.34E-22	3.24E-22	3.14E-22
-0.59799	3.90E-55	1.05E-35	8.83E-27	8.34E-22	7.50E-22	6.80E-22
-0.58794	9.46E-54	6.54E-35	3.03E-26	2.07E-21	1.72E-21	1.46E-21
-0.57789	2.20E-52	4.00E-34	1.02E-25	5.07E-21	3.93E-21	3.13E-21
-0.56784	4.92E-51	2.40E-33	3.43E-25	1.23E-20	8.89E-21	6.67E-21
-0.55779	1.06E-49	1.41E-32	1.13E-24	2.98E-20	2.00E-20	1.41E-20
-0.54774	2.18E-48	8.09E-32	3.69E-24	7.13E-20	4.46E-20	2.96E-20
-0.53769	4.31E-47	4.55E-31	1.19E-23	1.69E-19	9.87E-20	6.19E-20
-0.52764	8.20E-46	2.51E-30	3.78E-23	3.98E-19	2.17E-19	1.28E-19
-0.51759	1.50E-44	1.36E-29	1.19E-22	9.29E-19	4.74E-19	2.65E-19
-0.50754	2.63E-43	7.19E-29	3.68E-22	2.15E-18	1.03E-18	5.43E-19
-0.49749	4.43E-42	3.73E-28	1.13E-21	4.92E-18	2.21E-18	1.11E-18
-0.48744	7.16E-41	1.89E-27	3.41E-21	1.12E-17	4.72E-18	2.24E-18
-0.47739	1.11E-39	9.43E-27	1.02E-20	2.52E-17	1.00E-17	4.51E-18
-0.46734	1.66E-38	4.60E-26	3.01E-20	5.63E-17	2.11E-17	9.01E-18
-0.45729	2.38E-37	2.20E-25	8.76E-20	1.25E-16	4.40E-17	1.79E-17
-0.44724	3.28E-36	1.03E-24	2.52E-19	2.73E-16	9.12E-17	3.53E-17
-0.43719	4.33E-35	4.71E-24	7.15E-19	5.95E-16	1.88E-16	6.93E-17
-0.42714	5.50E-34	2.11E-23	2.01E-18	1.28E-15	3.83E-16	1.35E-16
-0.41709	6.71E-33	9.29E-23	5.55E-18	2.74E-15	7.77E-16	2.61E-16
-0.40704	7.86E-32	4.00E-22	1.52E-17	5.79E-15	1.56E-15	5.03E-16
-0.39698	8.84E-31	1.69E-21	4.10E-17	1.21E-14	3.12E-15	9.61E-16
-0.38693	9.54E-30	6.98E-21	1.09E-16	2.52E-14	6.18E-15	1.82E-15
-0.37688	9.90E-29	2.83E-20	2.87E-16	5.19E-14	1.21E-14	3.44E-15
-0.36683	9.86E-28	1.12E-19	7.45E-16	1.06E-13	2.37E-14	6.45E-15
-0.35678	9.44E-27	4.36E-19	1.91E-15	2.14E-13	4.58E-14	1.20E-14
-0.34673	8.68E-26	1.66E-18	4.82E-15	4.28E-13	8.79E-14	2.22E-14
-0.33668	7.66E-25	6.18E-18	1.20E-14	8.47E-13	1.67E-13	4.07E-14
-0.32663	6.49E-24	2.26E-17	2.96E-14	1.66E-12	3.16E-13	7.42E-14
-0.31658	5.28E-23	8.06E-17	7.18E-14	3.22E-12	5.92E-13	1.34E-13
-0.30653	4.13E-22	2.82E-16	1.72E-13	6.20E-12	1.10E-12	2.42E-13
-0.29648	3.10E-21	9.68E-16	4.06E-13	1.18E-11	2.02E-12	4.32E-13
-0.28643	2.24E-20	3.25E-15	9.45E-13	2.22E-11	3.70E-12	7.66E-13
-0.27638	1.55E-19	1.07E-14	2.17E-12	4.13E-11	6.69E-12	1.35E-12
-0.26633	1.03E-18	3.43E-14	4.90E-12	7.61E-11	1.20E-11	2.36E-12
-0.25628	6.58E-18	1.08E-13	1.09E-11	1.39E-10	2.14E-11	4.09E-12
-0.24623	4.04E-17	3.32E-13	2.39E-11	2.49E-10	3.76E-11	7.04E-12
-0.23618	2.38E-16	1.00E-12	5.17E-11	4.43E-10	6.57E-11	1.20E-11
-0.22613	1.35E-15	2.94E-12	1.10E-10	7.79E-10	1.14E-10	2.04E-11
-0.21608	7.32E-15	8.45E-12	2.29E-10	1.35E-09	1.95E-10	3.44E-11
-0.20603	3.82E-14	2.37E-11	4.71E-10	2.31E-09	3.30E-10	5.74E-11
-0.19598	1.91E-13	6.48E-11	9.49E-10	3.89E-09	5.54E-10	9.50E-11
-0.18593	9.18E-13	1.73E-10	1.88E-09	6.46E-09	9.20E-10	1.56E-10
-0.17588	4.23E-12	4.49E-10	3.64E-09	1.06E-08	1.51E-09	2.54E-10
-0.16583	1.87E-11	1.13E-09	6.90E-09	1.69E-08	2.45E-09	4.10E-10
-0.15578	7.88E-11	2.77E-09	1.28E-08	2.66E-08	3.92E-09	6.56E-10
-0.14573	3.18E-10	6.56E-09	2.31E-08	4.09E-08	6.21E-09	1.04E-09
-0.13568	1.22E-09	1.50E-08	4.05E-08	6.11E-08	9.68E-09	1.63E-09
-0.12563	4.43E-09	3.29E-08	6.85E-08	8.84E-08	1.49E-08	2.53E-09
-0.11558	1.52E-08	6.88E-08	1.11E-07	1.23E-07	2.24E-08	3.89E-09
-0.10553	4.85E-08	1.36E-07	1.70E-07	1.60E-07	3.32E-08	5.91E-09
-0.09548	1.42E-07	2.48E-07	2.41E-07	1.91E-07	4.81E-08	8.87E-09
-0.08543	3.70E-07	4.01E-07	2.95E-07	1.89E-07	6.78E-08	1.31E-08
-0.07538	7.93E-07	5.12E-07	2.58E-07	1.05E-07	9.25E-08	1.92E-08
-0.06533	1.04E-06	2.52E-07	-5.23E-08	-1.59E-07	1.21E-07	2.75E-08

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-0.05528	-1.61E-06	-1.37E-06	-1.03E-06	-7.82E-07	1.48E-07	3.88E-08
-0.04523	-1.93E-05	-6.97E-06	-3.49E-06	-2.08E-06	1.63E-07	5.34E-08
-0.03518	-9.61E-05	-2.28E-05	-9.00E-06	-4.58E-06	1.45E-07	7.17E-08
-0.02513	-0.00037	-6.29E-05	-2.05E-05	-9.19E-06	5.21E-08	9.30E-08
-0.01508	-0.00123	-0.00016	-4.32E-05	-1.73E-05	-1.89E-07	1.15E-07
-0.00503	-0.00368	-0.00036	-8.64E-05	-3.13E-05	-7.04E-07	1.33E-07
0.005025	-0.01003	-0.0008	-0.00017	-5.47E-05	-1.70E-06	1.37E-07
0.015075	-0.0252	-0.00168	-0.00031	-9.29E-05	-3.53E-06	1.08E-07
0.025126	-0.05869	-0.00337	-0.00055	-0.00015	-6.73E-06	1.23E-08
0.035176	-0.12713	-0.0065	-0.00096	-0.00025	-1.21E-05	-2.05E-07
0.045226	-0.2566	-0.01208	-0.00164	-0.0004	-2.11E-05	-6.35E-07
0.055276	-0.48314	-0.02169	-0.00274	-0.00063	-3.55E-05	-1.42E-06
0.065327	-0.84912	-0.03763	-0.00447	-0.00097	-5.82E-05	-2.80E-06
0.075377	-1.393	-0.06316	-0.00714	-0.00147	-9.36E-05	-5.10E-06
0.085427	-2.13216	-0.10262	-0.01117	-0.0022	-0.00015	-8.86E-06
0.095477	-3.0417	-0.16146	-0.01713	-0.00325	-0.00023	-1.48E-05
0.105528	-4.03682	-0.24607	-0.02578	-0.00473	-0.00035	-2.42E-05
0.115578	-4.969	-0.36332	-0.03806	-0.00679	-0.00053	-3.86E-05
0.125628	-5.6445	-0.51969	-0.05515	-0.00963	-0.00078	-6.03E-05
0.135678	-5.8663	-0.72008	-0.07844	-0.0135	-0.00115	-9.26E-05
0.145729	-5.48988	-0.96626	-0.10953	-0.01867	-0.00166	-0.00014
0.155779	-4.47383	-1.25521	-0.15017	-0.02551	-0.00238	-0.00021
0.165829	-2.90529	-1.5776	-0.20217	-0.03444	-0.00337	-0.00031
0.175879	-0.98771	-1.91683	-0.26726	-0.04593	-0.00472	-0.00045
0.18593	1.006437	-2.24899	-0.34689	-0.06053	-0.00653	-0.00065
0.19598	2.801716	-2.54409	-0.44206	-0.07882	-0.00895	-0.00092
0.20603	4.181792	-2.76858	-0.55301	-0.10142	-0.01213	-0.0013
0.21608	5.028548	-2.88918	-0.679	-0.12897	-0.01627	-0.00181
0.226131	5.330124	-2.87733	-0.81805	-0.16207	-0.02161	-0.0025
0.236181	5.161793	-2.71386	-0.96677	-0.20125	-0.02841	-0.00342
0.246231	4.651349	-2.39276	-1.12027	-0.24695	-0.03698	-0.00463
0.256281	3.94224	-1.92337	-1.27218	-0.29941	-0.04766	-0.00622
0.266332	3.16429	-1.33058	-1.41482	-0.35868	-0.06081	-0.00828
0.276382	2.416426	-0.65254	-1.53957	-0.42448	-0.07683	-0.01093
0.286432	1.761157	0.063469	-1.63736	-0.49622	-0.09611	-0.0143
0.296482	1.227679	0.766676	-1.6993	-0.57291	-0.11905	-0.01855
0.306533	0.819728	1.408775	-1.7174	-0.65314	-0.14602	-0.02385
0.316583	0.524782	1.949372	-1.6853	-0.73508	-0.17734	-0.03041
0.326633	0.322323	2.360014	-1.59897	-0.81647	-0.21325	-0.03845
0.336683	0.190015	2.626296	-1.45723	-0.89469	-0.25388	-0.04821
0.346734	0.107542	2.747902	-1.2621	-0.9668	-0.29924	-0.05995
0.356784	0.058442	2.736835	-1.01886	-1.02968	-0.34915	-0.07392
0.366834	0.030499	2.614342	-0.73588	-1.08013	-0.40325	-0.09039
0.376884	0.015285	2.407189	-0.42412	-1.11504	-0.46094	-0.10961
0.386935	0.007357	2.143942	-0.09642	-1.13153	-0.5214	-0.13182
0.396985	0.0034	1.851776	0.233404	-1.12718	-0.58355	-0.1572
0.407035	0.00151	1.554126	0.551514	-1.10011	-0.64605	-0.18591
0.417085	0.000644	1.269318	0.845021	-1.0492	-0.70735	-0.21801
0.427136	0.000264	1.010114	1.10284	-0.97418	-0.76569	-0.2535
0.437186	0.000104	0.783999	1.316364	-0.87568	-0.81916	-0.29225
0.447236	3.91E-05	0.593958	1.479901	-0.75528	-0.86574	-0.33404
0.457286	1.42E-05	0.439523	1.590846	-0.61547	-0.9034	-0.37849
0.467337	4.95E-06	0.317856	1.649596	-0.45959	-0.93016	-0.42509
0.477387	1.65E-06	0.224751	1.659233	-0.29166	-0.94419	-0.47318
0.487437	5.32E-07	0.155437	1.625033	-0.11625	-0.94391	-0.52193
0.497487	1.64E-07	0.105179	1.553864	0.061766	-0.92803	-0.57039
0.507538	4.86E-08	0.069651	1.453538	0.237427	-0.89572	-0.61746
0.517588	1.38E-08	0.045149	1.332186	0.405899	-0.84657	-0.66194
0.527638	3.78E-09	0.028652	1.197701	0.56271	-0.78074	-0.70253
0.537688	9.93E-10	0.017804	1.057289	0.70394	-0.69889	-0.7379
0.547739	2.50E-10	0.010833	0.917153	0.826381	-0.60225	-0.7667
0.557789	6.07E-11	0.006456	0.782306	0.927649	-0.49259	-0.78764
0.567839	1.41E-11	0.003768	0.656504	1.006252	-0.37213	-0.7995
0.577889	3.15E-12	0.002154	0.542285	1.061595	-0.24347	-0.8012
0.58794	6.76E-13	0.001206	0.441085	1.09395	-0.10955	-0.79188
0.59799	1.39E-13	0.000661	0.353408	1.104376	0.026536	-0.77089
0.60804	2.76E-14	0.000355	0.279011	1.0946	0.161602	-0.73786

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0.61809	5.24E-15	0.000187	0.217108	1.066885	0.292527	-0.69274
0.628141	9.56E-16	9.64E-05	0.166549	1.023874	0.416362	-0.6358
0.638191	1.68E-16	4.87E-05	0.125984	0.968435	0.530443	-0.56765
0.648241	2.82E-17	2.41E-05	0.093989	0.903511	0.632482	-0.48922
0.658291	4.56E-18	1.17E-05	0.069166	0.831991	0.720641	-0.40175
0.668342	7.08E-19	5.54E-06	0.050215	0.756587	0.793585	-0.30678
0.678392	1.06E-19	2.57E-06	0.035972	0.679756	0.850504	-0.20607
0.688442	1.51E-20	1.17E-06	0.025428	0.603626	0.891115	-0.10155
0.698492	2.08E-21	5.23E-07	0.01774	0.529966	0.915637	0.004722
0.708543	2.75E-22	2.29E-07	0.012216	0.460172	0.924749	0.110639
0.718593	3.49E-23	9.79E-08	0.008303	0.395271	0.919528	0.214117
0.728643	4.25E-24	4.11E-08	0.005571	0.335944	0.901374	0.313157
0.738693	4.97E-25	1.69E-08	0.00369	0.28257	0.871933	0.405916
0.748744	5.58E-26	6.79E-09	0.002413	0.235259	0.83301	0.490755
0.758794	6.03E-27	2.68E-09	0.001558	0.19391	0.786486	0.566293
0.768844	6.25E-28	1.03E-09	0.000993	0.158252	0.734247	0.631438
0.778894	6.22E-29	3.91E-10	0.000625	0.127894	0.678111	0.685413
0.788945	5.95E-30	1.45E-10	0.000389	0.102366	0.619776	0.727764
0.798995	5.46E-31	5.26E-11	0.000239	0.081155	0.560773	0.758357
0.809045	4.82E-32	1.87E-11	0.000145	0.063734	0.502437	0.777364
0.819095	4.08E-33	6.52E-12	8.65E-05	0.049586	0.445888	0.785236
0.829146	3.32E-34	2.23E-12	5.11E-05	0.038222	0.392027	0.782672
0.839196	2.59E-35	7.44E-13	2.99E-05	0.029193	0.341535	0.770572
0.849246	1.95E-36	2.44E-13	1.72E-05	0.022095	0.294889	0.749999
0.859296	1.40E-37	7.82E-14	9.80E-06	0.016571	0.252379	0.722125
0.869347	9.70E-39	2.46E-14	5.51E-06	0.012317	0.214132	0.688191
0.879397	6.45E-40	7.57E-15	3.06E-06	0.009074	0.180133	0.649455
0.889447	4.12E-41	2.28E-15	1.68E-06	0.006625	0.15026	0.607158
0.899497	2.52E-42	6.74E-16	9.08E-07	0.004795	0.124301	0.562485
0.909548	1.49E-43	1.95E-16	4.85E-07	0.00344	0.101983	0.516535
0.919598	8.41E-45	5.53E-17	2.56E-07	0.002446	0.082994	0.470301
0.929648	4.56E-46	1.54E-17	1.34E-07	0.001725	0.066998	0.424652
0.939698	2.38E-47	4.18E-18	6.89E-08	0.001205	0.053654	0.380324
0.949749	1.19E-48	1.11E-18	3.50E-08	0.000835	0.042629	0.337918
0.959799	5.74E-50	2.91E-19	1.76E-08	0.000574	0.033605	0.297901
0.969849	2.65E-51	7.44E-20	8.73E-09	0.000391	0.026285	0.260612
0.979899	1.18E-52	1.86E-20	4.28E-09	0.000264	0.020401	0.226272
0.98995	5.01E-54	4.57E-21	2.07E-09	0.000177	0.015713	0.194998
1	2.05E-55	1.10E-21	9.90E-10	0.000117	0.01201	0.166815

Figure 1 represents visually the Gaussian Difference Continuous Distribution (GDCD) error propagation across six different numerical solvers. On the x-axis, the numerical error magnitude is represented, and the y-axis is the probability density function (PDF), which shows how error values fluctuate probabilistically under various solver scenarios.

The curves of Newton's Method and Runge-Kutta Solver indicate that there is considerable sharpness, meaning step-size fluctuations are controlled well, which, in turn, leads to faster convergence and better stability. GDCD-based error adjustments benefited these solvers of which the outcome is a reduction in oscillations and overshooting risks.

On the other hand, Monte Carlo Simulations and the Numerical Integration methods reveal long-tailed distributions, suggesting that errors fluctuate over the entire simulation period. This behavior is a hint that the use of probabilistic models is necessary not only for uncertainty quantification but mainly in the area of random sampling and function approximations.

Finite Difference Methods and Differential Equation Solvers show a behavior that evolves slowly, common examples are gradual transitions in probability distributions, in the form of intermediate behaviors. The patterns imply that an error is accumulating over the successive computational steps with the need of probabilistic step-size refinement to preserve solver accuracy.

Taken as a whole, the information contained in Figure 1 verifies that the GDCD model of error provides a more exhaustive error modeling approach vis-a-vis traditional single-Gaussian-based ones. This permits more accurate quantification of uncertainty, optimal step-size, and enhancement of the solver's stability, the result of which is a very significant advantage in the area of scientific computing, numerical optimization, and real-time engineering simulations.

Through GDCD-based probability visualizations, numerical solvers can attain higher precision, increased stability, and adaptive performance, which may lead to obtaining more accurate results in complex computational environments.

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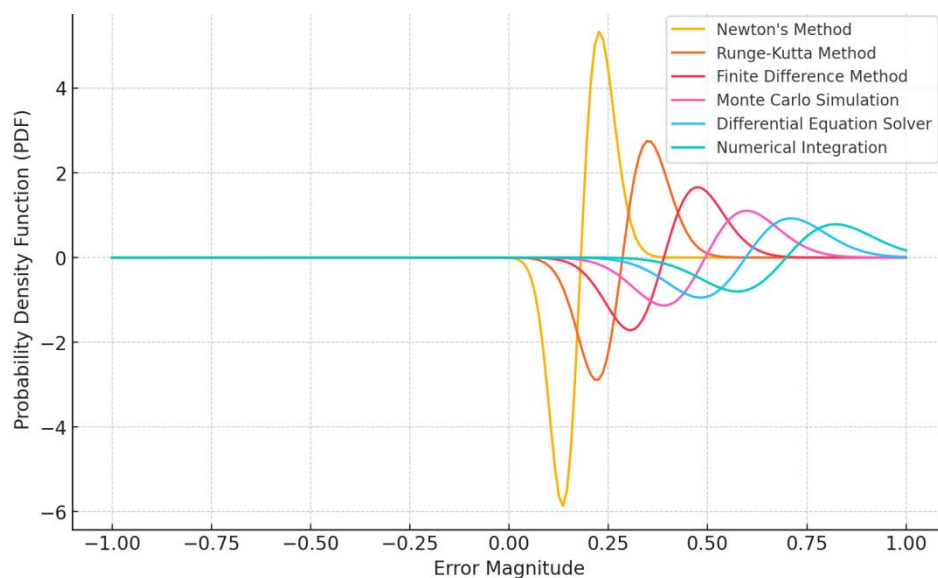


Figure 1: Analytical Gaussian difference function in numerical methods

CONCLUSION

This study establishes the fact that the Gaussian Difference Continuous Distribution (GDCD) is a more reliable framework for the purposes of modeling numerical error propagation in scientific computing, engineering, and applied mathematics. Employing GDCD to six basic numerical methods (Newton's method, Runge-Kutta solvers, finite difference methods, Monte Carlo simulations, differential equation solvers, and numerical integration techniques), we observe that one-step stochastic process is noticeably better in solver stability, accuracy, and error estimation under certain conditions than deterministic counterparts.

The evidence reveals that the GDCD-based models are better on average than the traditional single-Gaussian error assumptions if the numerical errors are the result of two competing sources of variability. For example, Newton's method exhibits better behavior as the number of iterations increases, Monte Carlo simulations provide a more accurate estimate of the uncertainty. The GDCD-based step-size adjustments in the finite E-difference and numerical E-integration methods lead to a higher accuracy in the function approximation and imply less truncation errors.

One of the most important characteristics of the GDCD-based numerical solvers is that they are able to dynamically update step sizes and error estimates based on the statistical feedback in real-time. Contrary to the traditional version, which divides errors into fixed parts, GDCD allows probabilistic occurrences to enter the numerical calculations, hence this diminishes the possibility of an unsolvable equation, a divergent calculus, or an overshooting.

These findings shed a light on GDCD which can be possibly used in the areas of high-precision numerical applications, e.g., machine learning optimizers, uncertainty modeling, and high-performance simulations. Future research is suggested to be focused on the application of GDCD into adaptive solvers and real-time computational frameworks thus enhancing numerical precision, efficiency, and robustness regardless of scientific disciplines.

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