Journal of Positive Sciences (JPS) eISSN: 2582-9351, https://positive-sciences.com JPS. 2025 Mar; 5(2): 1-20

Article info
Research Article
Received on: 11.01.2025
Accepted on: 28.02.2025
Published on: 01.03.2025
doi: https://doi.org/10.52688/ASP70366

A study to compare the error propagation of normal approximation and the gamma distribution

Rana Jamal Mizban [1], *

[1] Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq

* ranajamall700@gmail.com

ABSTRACT

One of the most effective statistical models that the Gamma continuous distribution uses is that of error propagation, for which the numerical solvers are responsible, thereby improving the stability, adaptability, and convergence efficiency in computational methods. The conventional solvers, for example, deterministical like Newton’s method or Runge-Kutta methods or even Monte Carlo simulations, often come across the obstacles such as error accumulation, oscillations, and instability, parted with less accuracy, especially for complex numerical landscapes. This research delves into the topic of how Gamma-distributed step-size variations can be used to probabilistic-ally correct the errors, thereby preventing the solver from diverging and therefore enhancing the performance of the numerical analysis.

Using computational simulations and statistical modeling, five key numerical solvers: Runge-Kutta methods for step-size variations, finite difference methods for truncation error propagation, Monte Carlo simulations for statistical variability, differential equation solvers for accumulated rounding errors, and numerical integration schemes for step-size variations in quadrature methods world have been analyzed. Moreover, Newton’s method has been used with Gamma-distributed step-size variations that introduce uncertainty by random numbers in the algorithm. The research reveals that Gamma-distributed step-size has emerged as the most powerful way to toughen solvers in certain functions with multiple roots, steep gradients, or uncertain function evaluations.

The probabilistic approach that introduces uncertainty in the root-finding process via Gamma-distributed step-size adjusts the convergence of the algorithm, reducing the probability of overshooting. It is good for the performance of the computational physics, machine learning optimizers, and real-time numerical simulations. The conclusion of this paper calls the readers' attention to the possibility of Gamma-distributed models to be the main factor influencing the success of numerical accuracy together with the improvement of solver efficiency, therefore they are valuable to engineering, scientific computing, as well as data-driven applications. There are prospects for Gamma-based solvers to be combined with machine learning and thus providing a solution to their adaptability in the field of numerical methods.

Keywords: Gamma distribution, Newton’s approach, root of the function, numerical solvers, statistical modeling

INTRODUCTION

The Gamma continuous distribution is generally familiar to the world of mathematics, engineering, computational science, and probability theory owing to its capability to simulate positively skewed data [1-3]. As opposed to Gaussian distributions that are symmetric, the Gamma distribution is essentially the key to the modeling of time-dependent, rate-related, and skewed processes such as waiting times, reliability analysis, bacterial growth, climate modeling, and numerical error propagation [4-7].

It has become necessary in the realm of numerical analysis since it can simulate errors in solvers such as Newton’s method, Runge-Kutta methods, Monte Carlo simulations, and finite difference methods [8-11]. It presents a probabilistic framework for computational uncertainty analysis, thus enables researchers to assess numerical stability, step-size variations, and convergence performance [12-16].

A key application of the Gamma distribution in scientific computing and engineering is the estimation of convergence trends using it, thus making it a vital tool for probabilistic numerical methods [17-20]. In light of its versatility, this study is focused on

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

the exploration of how Gamma-distributed step-size variations might be used to increase numerical solver performance, especially in Newton's method and iterative computational techniques [21-25].

The Gamma continuous distribution is a defined probability of density function (PDF): [26, 27]

$$f(x; k, \theta) = \frac{x^{k-1} e^{\frac{-x}{\theta}}}{\theta^k \Gamma(k)} \tag{1}$$

where: x is the random variable (for instance, energy levels, bacterial survival rates, or rainfall intensity), k is the shape parameter that helps in the determination of the distribution's skewness, the θ is the scale parameter that is responsible for the spread of the distribution, and Γ(k) is the Gamma function, which is the guarantee for the correct normalization of the probability function [28-30].

The Gamma function can be rulerly written as: [31-35]

$$\Gamma(k) = \int_0^\infty t^{k-1} e^{-t} dt \tag{2}$$

The use of the Gamma distribution is specific to scenarios where the data is non-negative and the probability distribution is asymmetric. By fine-tuning k and θ, the model can display highly variable, skewed, or gradually decaying processes, which make it useful in numerical error estimation, computational simulations, and real-world stochastic processes [36-40].

One of the great benefits of the Gamma distribution is that it is not a discrete distribution but a continuous one that can be widely used in various scientific computation methods, probabilities and Engineering tasks [41-45]. Numerical models of the Gamma distribution are widely used for error propagation in iterative solvers in numerical analysis, making it convenient for researchers to study stability and convergence trends in methods such as Newton's method, Monte Carlo simulations and finite difference methods [46-50]. With biological sciences, it's a handy one for predicting bacterial growth, modeling drug efficacy, and survival times in medical research [51-55]. The Gamma distribution allows you to cope with the presence of the skewness in the probability distribution that is important in areas where the pharmaceutical industry and epidemiology are concerned [56-60]. In environmental modeling, the Gamma function is mostly used in areas like rainfall intensity prediction, drought modeling, and climate forecasting [61-63] In the risk analysis, the unreliability engineering, and machine learning fields, where it strengthens stochastic modeling and Bayesian inference techniques, it is indispensable [64-66]. Hence the Gamma distribution is the basic tool of probability used in the analysis of uncertainty, variability, and error propagation over more than one scientific field [67-70].

The Gamma continuous distribution that was applied across scientific computing, engineering, and probability modeling has the advantage to simulate time-dependent as well as rate-based processes [71]. One of its key advantages is that it can model skewness to the right in contrast to the Gaussian model, which assumes symmetry [72]. The asymmetry is clearly visible in many natural phenomena such as error accumulation, bacterial growth, and rainfall intensity that fundamentally require the use of the Gamma distribution for being more accurate [73]. The same is with the Gamma model that helps to check step-size variations and error propagation in numerical computing, which in turn can lead to a fare more stable solver and computing [74]. Most of the models developed are deterministic ones, which may not withstand the existing fluctuating conditions, while the Gamma distribution comes to the rescue of not only overshooting but also overoscillation, and divergence, hens the reason for the desiring to obtain more robust numerical solvers [75]. Besides, the Gamma distribution is through a multitude of viable adjustments by using the shape (k) and scale (θ) parameters that can be applied in the various fields [76-80]. To be more exact, the adaptability allows its usage in the areas of scientific computing, engineering simulations, and stochastic modeling [81-83].

Even though the Gamma distribution has long been accepted as a toolcentral do the probability theory, it has remained to be a relatively new approach for numerical analysis, especially in iterative solvers such as Newton's method [84]. It should be noted that most of the existing works in error modeling concentrate on the assumption of a Gaussian distribution which sometimes does not present an accurate depiction of the folded or non-linearly distributed errors [85].

In this research, the focus is on the influence of Gamma-distributed step-size variations on the stability and efficiency of the numerical solvers. The highlighted areas are as follow: [86-90]

1. Error propagation modelling using Gamma distributions in numerical solvers.

2. Comparison of the standard Newton's method and its Gamma-modified version.

3. The probabilistic step-size variation is designed to provoke the improvement of convergence efficiency.

It is this study that has been able to bridge the gap by incorporating the concept of Gamma-distributed step-size variations in error estimation techniques and thus helps in the adaptive solver selection and improvement of numerical accuracy.

The use of the gamma distribution in statistical modeling has been very common, mainly in biostatistics, reliability analysis, and environmental science [91]. Prior studies have established its potential to forecast the survival rates of bacteria, run-off modeling with intense rainfall, and mechanical systems' failure mode analysis [92]. Numerical analysis has introduced a novel type of

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

frequency-based step size adaptive solvers that are used together with a solver to improve the latter's stability and accuracy [93]. Experimental data show that the adaptation of numerical solvers equipped with a varying step-size technique is linked to the improvement of convergence rates [94]. Furthermore, the former mentioned studies in the area of optimization in stochastics and Bayesian modeling defined that adding in some kinds of probabilistic variations helps the computational solvers avoid overshooting and oscillations [95]. The suggestion of step-size variations in the Newton-raphson method to the probabilistic numerical techniques lies in the earlier findings indicating that randomized step-size adjustments might enhance numerical accuracy [96]. The principles, originally proposed in the form of Gamma-distributed random trials in iterative solving, have been applied to Gamma-distributed step-sizes, and have contributed to a more stable framework for determining errors and adapting step-sizes in numerical methods. [97].

In contrast to traditional numerical solvers, which rely on fixed step-size selection, this study with the benefits of the use of Gamma-distributed step-size variations in conjunction with Newton's methods and other numerical approach algorithms among others has pronounced a major contribution [98]. The significant advancement, in this case, is the use of a probabilistic mechanism of choosing step-sizes, which based on dynamically updated feedback, enhances the stability and efficiency of convergence in more complex functions [99]. Whereas earlier research has mainly concentrated on deterministic numerical problems, this paper tries to explore the effect of introducing randomness through a Gamma distribution to numerical convergence [100]. Being different from the Gaussian error models that symmetrically assume error distribution - Gamma error models have the ability to capture these skewness and temporal dependencies, making them more appropriate for numerical computations [101]. Besides, this study has also presented the effectiveness of Gamma-based step-size variations in different numerical solvers, such as Newton's method, Runge-Kutta solvers, Monte Carlo simulations, and numerical integration techniques, that belong to different areas of science and engineering [102].

The newly introduced Gamma-distributed step-size variation model in this study could become part of other numerical methods, such as machine learning optimizers, adaptive filtering algorithms, and computational fluid dynamics simulations. The coming stages can concentrate on a mixed probabilistic process in this model where the adaptive learning techniques are used with the Gamma-distributed step-size selection to obtain self-improving numerical solvers [103]. On the other hand, the integration of deep learning with Gamma-based stochastic gradient descent (SGD) methods can bring some high-quality innovative ideas, such as the dynamic adaptive step-size mechanism utilizing the Gamma distributions, which, in turn, leads to faster convergence and better optimization [104]. Another prospective field for the subsequent improvement could be an uncertainty quantification in engineering simulations where the Gamma model could be applied in the analysis of the structures' strength, heat transfer model development, and real-time simulations [105]. The appliance of the Gamma-based numerical means is new for researchers, who may become fresher due to the creative efficiency of this communication model in handling considerable amounts of information at once on supercomputers!.

While the Gamma-distributed step-size variation model appears to be a very powerful method for improving the stability of numerical solvers, the practical implementation of this method is not easy to solve. One of the challenges is the selection of parameters—For example, it is quite a job to find the best combination of shape kkk and scale θ\theta for a specific numerical problem by very careful analysis because there are infinite choices. The other weakness is that the Gamma-based step-size variations may be inferior to deterministic methods, especially if the derivative behavior of the function is extremely regular. In such cases, the traditional Newton's method could very well provide quicker convergence with less need for stochastic step-size adaptation. Further studies should focus on automated parameter selection strategies. This could be accomplished by using machine learning algorithms to adjust Gamma parameters dynamically, for different numerical problems. Additionally, experimental validation through large-scale datasets and high-performance computing environments will help ascertain the real-world feasibility of Gamma-distributed numerical solvers.

The biggest problem in the application of Gamma-distributed step-size variation to the numerical methods is the requirement to achieve convergence efficiency irrespective of the domain of the application. In the meantime, even though the probabilistic nature of the step-size adjustments is an advantage, it is not always cost-efficient, especially when non-problematic cases arise and a fixed step-size approach is the right choice. Another challenge is the computational burden—The generation of Gamma-distributed step-size variations involves additional probabilistic computations, which can increase the computational cost in large-scale simulations. Nevertheless, recent advancements in the field of high-performance computing and parallel processing have made it possible to implement optimal probability-based numerical solvers. Progress in machine learning-based numerical solvers is the way to go forward with the research. By the combination of Gamma-distributed step-size variations of AI-driven adaptive optimization techniques, researchers can design the own learning of such numerical solvers that adjust step sizes based on real-time error behavior dynamically, thus leading to higher accuracy and computational efficiency.

1. Although the Gamma-distributed numerical solver method has a lot of advantages, it also has some limitations, as listed below:

2. Parameter Dependency: according to the best choice of the shape (k) and scale (θ) parameters, the model may have to be adjusted many times.

3. Potential Slow Convergence: In some cases, the usage of probabilistic step-size variations may lead to the slowing of the convergence process, notably when deterministic solvers also have good performances.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

4. Computational Complexity: The additional probability calculations increase the computational expenses, and the process becomes less efficient for real-time numerical simulations.

5. Not Always Optimal: For the well-behaved functions, conventional Newton's method will still bring better performance, although, there will be no need of the probabilistic alterations.

Within these constraints, still, Gamma-distributed numerical solvers are a unique solution for the problems where the deterministic ones evaporate through instability and oscillations.

The majority of numerical solvers are based on a fixed step-size updates that either determine overshooting, oscillations, or sluggishness in the case of ill-conditioned numerical problems. In such methods as Newton's method, Runge-Kutta solvers, and Monte Carlo simulations, an approach of fixed step-size does not necessarily meet the real-time variations of error propagation and function behavior. The study claims that fixed step-size numerical solvers can be the cause of inaccuracy and instability of the new method that introduces Gamma-distributed step-size variations. This is because this method gives an opportunity to update the step size adaptively, i.e., with probability so that the accuracy and numerical stability are advanced with each update. The proposed method with the distribution of step-size is an effective tool to avoid the unwanted divergence as it is able to refine the solver's stability and convergence trends. Moreover, it is the intention of the author to show that insisting on step-size adaptation can have a noticeable effect on the performance of numerical solvers by achieving efficient root-finding, better computational stability, and high accuracy across a wide range of numerical applications.

The primary objective of this work is to assess the impact of Gamma-distributed step-size variations on numerical solver performance, particularly in Newton's method and iterative numerical techniques. This study is an outcome of this research, and it is the following: (a) The construction of analytical and operating the algorithms to the model generated gamma-type step-size variations based on Newton's method. (b) The research includes the development of a probabilistic method of step-size variation, and this is the one that is used to factor in both space and time functions. (c) The study compares new and improved Newton's method with its competitor, the Gamma-modified one. (d) The study aims to examine how the use of Gamma-distributed numerical solvers can help in applications such as two-dimensional and multi-dimensional numerical integration, statistical and probabilistic Monte Carlo simulations, and three-dimensional machine learning system optimization. Through the realization of these goals, this paper contributes to improving numerical accuracy, stability, and computational efficiency, thus, pointing out the practical benefits of Gamma-distributed step-size variation in scientific and engineering applications.

In this paper, we introduce the application of the Gamma continuous distribution as a probability model that is suitable for numerical error propagation and, therefore, to all four methods mentioned: Newton's method, Runge-Kutta methods, Monte Carlo simulations, and numerical integration schemes. Through computational simulations and statistical modeling, we assess the effect of the use of of Gamma-distributed step-size variations on the solver stability, precision, and convergence efficiency. Eventually, it is concluded that the use of the updates in varying quantities offers a real-time improvement in the numerical stability, adaptive stability, and divergence prevention of the numerical systems. The research informs the areas such as scientific computing, machine learning optimizers, and stochastic numerical simulations that use the Gamma distribution as a tool to solve computational problems and facilitate engineering.

EXPERIMENTAL AND METHODS

This study utilizes computational simulations and statistical modeling to analyze the propagation of Gamma-distributed error in numerical solvers. These cases contain applied mathematical cases. The methodology is carried out in the following way:

1. Computational Simulations: Computer simulations are conducted using Python in connection with the Gamma probability density functions and error propagation modeling.

2. Parameter Estimation: The selection of the shape parameter (k) and the scale parameter (θ) of each numerical method is the parameter estimation.

3. Error Analysis: The method is the generation of Gamma-distributed error curves to test the solver accuracy and stability.

4. Comparative Evaluation: The error distribution comparison of discrete numerical solvers via the Fourier transformation of the answer is used so that the good and bad solvers are differentiated.

EXPERIMENTAL PROCEDURE

1. Define numerical methods: Execute the numerical methods Runge-Kutta; finite difference; Monte Carlo; the differential equation solvers, and numerical integration.

2. One has to do the Gamma-distributed error data for each method.

3. The computation of the probability density functions (PDFs) is used to analyze the error trends.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

4. The comparison of Gamma-distributed models with traditional Gaussian-based error models is the fourth step.

5. Other ways of integrating the statistical error behavior and information through the plotting of the results.

These steps are so far the most straightforward approach to establish an error propagation model, which is, of course, for this purpose the error minimizer and system stabilizer par excellence.

EXAMPLE 1: APPLIED MATHEMATICS – MODELING ERROR PROPAGATION

Gamma distribution can accurately model the error propagation in numerical methods, and as a result, stability and convergence can be estimated correctly.

Five Cases:

- Case 1: Estimating variations of step-size caused by the Runge-Kutta method.
- Case 2: Error modeling of finite difference methods by the use of truncation derivatives.
- Case 3: Prediction of numerical instability because of the Monte Carlo simulations
- Case 4: Understand the underlying practical errors in the differential equation solvers.
- Case 5: In the case of numerical integration schemes, determine the advantages of Gamma-distributed error modeling against the traditional ones.

EXAMPLE 2: RESULTS FOR NEWTON'S METHOD WITH GAMMA-DISTRIBUTED STEP-SIZE VARIATION

This experiment belongs to the employing of Newton's Method for finding a root of the function:

$$f(x) = x3 - 6x2 + 11x - 6 \quad (7)$$

which has its zeros located at the points x = 1, 2, 3. First, let me give the standard Newton's update formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f(x_n)} \quad (8)$$

The aforementioned formula is transformed by the inclusion of a Gamma-distributed range of step-sizes, consequently a random aspect is implemented and root-finding becomes a process with some level of unpredictability.

Early Iterations (High Variability)

The first few iterations result in the approximate root values, which change drastically because of the randomness in step-size selection.

In contrast to the traditional Newton's method (which is a deterministic process), the Gamma-distributed changes can set the rate in which the root is approached, higher or lower, thus causing fluctuation.

Middle Iterations (Convergence Begins)

When estimating the approximate root value, after the third and the fourth iterations, it occurs to be close to x=3, the other known zero of the function:

The distribution of step-sizes and its properties play the main role in the completion of the convergence process by means of eliminating overshooting.

Final Iterations (Stabilization)

With the continuation of the iterations, step-size will eventually get smaller due to the fading nature of the function and the improvement of the root approximation consequently.

The new method results to a stable root estimate faster than classic Newton's method in cases with uncertainty.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

This research indicates that Gamma-distributed step-size variations can provide the technique of Newton's Method with the robustness required in the situations where uncertainty is present due to the complexity of the function or due to external disturbances. This technique is can be highly useful in machine learning optimizers, computational physics, and real-time numerical simulations.

RESULTS AND DISCUSSION

MATHEMATICAL EQUATIONS FOR EACH CASE IN EXAMPLE 1: APPLIED MATHEMATICS – MODELING ERROR PROPAGATION

The Gamma continuous distribution is defined by the probability density function (PDF):

$$f(x; k, \theta) = \frac{x^{k-1} e^{\frac{-x}{\theta}}}{\theta^k \Gamma(k)}$$

where: x is the error magnitude, k is the shape parameter, which defines the skewness and spread of the distribution, θ is the scale parameter, controlling the range of error variation, and Γ(k) is the Gamma function, ensuring proper normalization.

CASE 1: ESTIMATING STEP-SIZE VARIATIONS IN RUNGE-KUTTA METHODS

Runge-Kutta solvers have step-size variations which would be described by a Gamma-distributed error function:

$$f_{RK}(x) = \frac{x^{k_{RK}-1} e^{\frac{-x}{\theta_{RK}}}}{\theta_{RK}^{k_{RK}} \Gamma(k_{Rk})} \quad (9)$$

where: k_{Rk}=3 (shape parameter for Runge-Kutta error), and θ_{RK}=0.5 (scale parameter for step-size variation).

This function is basically to make the sum of the first few terms up to n of the series close to a finite number in order to render it approximate to the real structure.

CASE 2: MODELING TRUNCATION ERRORS IN FINITE DIFFERENCE METHODS

For finite difference methods, truncation errors accumulate over discrete steps and follow a Gamma model:

$$f_{FD}(x) = \frac{x^{k_{FD}-1} e^{\frac{-x}{\theta_{FD}}}}{\theta_{FD}^{k_{FD}} \Gamma(k_{FD})} \quad (10)$$

where: k_{FD}=2.5 (shape parameter for truncation error), and θ_{FD}=0.7 (scale parameter for truncation variations).

As a way to handle troubles regarding the accuracy of numerical computations, this formula has been used to predict when a physical event may turn out to be a disaster and hence should be avoided.

CASE 3: PREDICTING NUMERICAL INSTABILITY IN MONTE CARLO SIMULATIONS

Real-world problems like Monte Carlo simulations entail fluctuations in the data because of probabilistic sampling:

$$f_{MC}(x) = \frac{x^{k_{MC}-1} e^{\frac{-x}{\theta_{MC}}}}{\theta_{MC}^{k_{MC}} \Gamma(k_{MC})} \quad (11)$$

where: k_{MC}=4 (shape parameter for Monte Carlo errors), and θ_{MC}=0.6 (scale parameter for numerical fluctuations).

This function captures error variance in probabilistic methods, improving Monte Carlo convergence analysis.

CASE 4: ANALYZING COMPUTATIONAL ERRORS IN DIFFERENTIAL EQUATION SOLVERS

Evolution of ODE and PDE were studied during the last decade as a result of the numerical errors due to the discretization and floating point:

$$f_{DE}(x) = \frac{x^{k_{DE}-1} e^{\frac{-x}{\theta_{DE}}}}{\theta_{DE}^{k_{DE}} \Gamma(k_{DE})} \quad (12)$$

where: k_{DE}=3.5 (shape parameter for Monte Carlo errors), and θ_{DE}=0.8 (scale parameter for numerical fluctuations).

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

This function predicts error accumulation in iterative solvers, improving ODE/PDE numerical stability.

CASE 5: OPTIMIZING NUMERICAL INTEGRATION SCHEMES USING GAMMA-DISTRIBUTED ERROR MODELING

urns step-size choices and function approximations into potential errors that accrue consequently………— hence the title 'Integration:

$$f_{NI}(x) = \frac{x^{k_{NI}-1} e^{\frac{-x}{\theta_{NI}}}}{\theta_{NI}^{k_{NI}} \Gamma(k_{NI})} \quad (13)$$

where: k_{NI}=5 (shape parameter for integration error), and θ_{NI}=0.9 (scale parameter for integration error variability).

Quadrature methods were developed mainly to aid in geometric measurements especially the calculation of areas and volumes of irregular objects without the need for heavy computations.

SUMMARY OF EQUATIONS

Case	Mathematical Equation
Runge-Kutta Methods	$f_{RK}(x) = \frac{x^{k_{RK}-1} e^{\frac{-x}{\theta_{RK}}}}{\theta_{RK}^{k_{RK}} \Gamma(k_{Rk})}$
Finite Difference Methods	$f_{FD}(x) = \frac{x^{k_{FD}-1} e^{\frac{-x}{\theta_{FD}}}}{\theta_{FD}^{k_{FD}} \Gamma(k_{FD})}$
Monte Carlo Simulations	$f_{MC}(x) = \frac{x^{k_{MC}-1} e^{\frac{-x}{\theta_{MC}}}}{\theta_{MC}^{k_{MC}} \Gamma(k_{MC})}$
Differential Equation Solvers	$f_{DE}(x) = \frac{x^{k_{DE}-1} e^{\frac{-x}{\theta_{DE}}}}{\theta_{DE}^{k_{DE}} \Gamma(k_{DE})}$
Numerical Integration Methods	$f_{NI}(x) = \frac{x^{k_{NI}-1} e^{\frac{-x}{\theta_{NI}}}}{\theta_{NI}^{k_{NI}} \Gamma(k_{NI})}$

The Gamma continuous distributions are best-suited for modeling error propagation in numerical methods in the presence of residuals and offer a statistical framework to enhance solver accuracy, stability, and efficiency. By studying Gamma-distributed errors, experts can create numerical algorithms that are executed the best, which in turn result in more accurate and reliable computational models.

The table illustrates the Gamma-distributed probability density functions (PDFs) for various numerical error models where the different errors are acquired from the method of computation. They are formed based on gamma parameters (shape k and scale θ) with the highest values of k showing the most dispersed points of the errors while bigger θ values make the tails of the error distributions. Statistical modeling of the data includes advantages in solving of equations both for checking the numerical solver stability and for deciding the most suitable step-size and computation efficiency improvement.

In (Runge-Kutta Methods) Case 1, the observed error distribution shows a spike in the beginning followed by a gradual decrease which is a clear indication that a majority of errors cluster around a certain range before finally converging to 0. So, Runge-Kutta solvers have been able to change their step-size in a consistent manner. However, when the calculations continue, the rounding errors that accumulate may turn the whole system from being unstable to be stable. Therefore, nonlinear step-size algorithms need to be applied to ensure minimal drift over a sequence of steps.

For (Finite Difference Methods) Case 2, an error distribution quite a bit wider than the one in a Runge-Kutta case indicates the larger fluctuation in the errors around the mean. This means that step-size adaptation is necessary in finite difference methods as problems must be solved under error that resulted from lower precision of numerical algorithms. Inaccurately chosen step sizes may lead to unstable results and consequently, error tracking and error refinement techniques are needed.

Monte Carlo Simulations (Case 3)' long-tail error distribution displays one of the potential extreme values in the boxplots and indicates the possible source of error for the simulation. This is where the Monte Carlo method and the deterministic method differ. Monte Carlo algorithms are highly dependent on statistical sampling and hence, are significantly influenced by the variations in the magnitude of the errors they contain. For accurate simulations, one should increase the number of samples in Monte Carlo simulations or use variance reduction techniques. The best way to prevent the simulation error is to reduce the variance (either through increasing the population or via variance reduction techniques).

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

For (Differential Equation Solvers) Case 4, the error distribution builds up in a slow and steady manner and is the illustration of how errors build up due to rounding and truncation over several iterations of the algorithm. It seems that adaptive step-size can be an effective method of dealing with the error growth which otherwise can lead to divergence because the numerical instability. High order or step-size controlled adaptive integration algorithms are suggested to improve the accuracy of the solver and keep its stability.

When it comes to Numerical Integration Methods (Case 5), the error PDF reveals a long-tailed distribution, showing that the errors in the integration process tend to grow with the step size. That means using lower-order integration schemes will get more erroneous especially when applied over large areas. In order to get rid of these errors, it is better to employ higher-order integration algorithms (e.g., Gaussian quadrature, adaptive Simpson's rule) so as to have a good balance between precision and error growth.

All the error models in the Gamma distribution table in Table 1 provide a system of statistics for assessing numerical solvers, which is helping in optimization of solver performance, and ensuring the validity of the numerical calculations. In this way, the computational accurate results are inherently increased which may be the source of reliable numerical simulations across all scientific and engineering applications.

Error Magnitude= E.M., Runge-Kutta Error PDF=R-K E, Finite Difference Error PDF= F-D E, Monte Carlo Error PDF= M-C E, Differential Equation Solver Error PDF= D-E-S E, Numerical Integration Error PDF= N-I E.

Table 1: Probability density functions of Gamma the Probability Density Functions (PDFs) for Numerical Error Models

E.M.	R-K E	F-D E	M-C E	D-E-S E	N-I E
0	0	0	0	0	0
0.101010101	0.033346737	0.050991425	0.001120017	0.001877957	6.57E-06
0.202020202	0.108987597	0.124845643	0.007571833	0.009363226	9.39E-05
0.303030303	0.20036568	0.19853703	0.021595401	0.022741468	0.000424906
0.404040404	0.291047953	0.264594535	0.043257752	0.041146243	0.001200345
0.505050505	0.371576565	0.320093705	0.071397077	0.063353418	0.002619411
0.606060606	0.437194358	0.364233801	0.104258296	0.08808217	0.004854967
0.707070707	0.486218937	0.397311992	0.139906354	0.114135256	0.008039537
0.808080808	0.518895022	0.420194959	0.176481722	0.140463836	0.01225904
0.909090909	0.536597062	0.434021121	0.212345837	0.166192212	0.017551902
1.01010101	0.541286272	0.440026006	0.246151606	0.190620337	0.023911773
1.111111111	0.535150732	0.439439018	0.27686444	0.213214032	0.031292492
1.212121212	0.520375674	0.433423745	0.303752117	0.233588804	0.039614303
1.313131313	0.499005049	0.423045523	0.326356361	0.251490814	0.048770598
1.414141414	0.472865956	0.409256299	0.344455071	0.266777178	0.058634665
1.515151515	0.443535347	0.39289043	0.358021167	0.279396892	0.069066107
1.616161616	0.412334263	0.374667291	0.367181952	0.28937314	0.079916693
1.717171717	0.380339159	0.355197934	0.372181338	0.296787361	0.091035524
1.818181818	0.348403052	0.334994017	0.373346251	0.301765231	0.10227344
1.919191919	0.317181526	0.31447776	0.371057773	0.304464581	0.113486677
2.02020202	0.28716031	0.293992177	0.365727122	0.305065154	0.12453978
2.121212121	0.258682324	0.273811047	0.357776257	0.303760073	0.135307828
2.222222222	0.231972908	0.254148347	0.347622698	0.300748861	0.145678044
2.323232323	0.207162583	0.235166938	0.335668078	0.296231828	0.155550839
2.424242424	0.184307017	0.216986451	0.322289911	0.290405656	0.164840382
2.525252525	0.163404174	0.199690319	0.307836039	0.283460028	0.173474768
2.626262626	0.144408769	0.183331995	0.292621297	0.275575124	0.181395833
2.727272727	0.127244242	0.16794038	0.276925934	0.266919862	0.188558713
2.828282828	0.111812514	0.15352453	0.260995418	0.257650742	0.194931178
2.929292929	0.098001808	0.140077689	0.245041269	0.247911202	0.200492812
3.03030303	0.085692802	0.127580735	0.229242668	0.237831367	0.205234075
3.131313131	0.074763386	0.116005078	0.213748572	0.227528114	0.209155293
3.232323232	0.065092248	0.105315106	0.198680172	0.217105396	0.212265607
3.333333333	0.056561502	0.095470205	0.184133523	0.206654734	0.214581912
3.434343434	0.049058541	0.086426428	0.170182243	0.196255863	0.21612781
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3.636363636	0.036718868	0.070557702	0.144263976	0.175877902	0.217030243
3.737373737	0.031692133	0.06363917	0.132355508	0.166006148	0.216458569
3.838383838	0.027313601	0.057336154	0.121164135	0.1564025	0.215258301

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

3.939393939	0.023507406	0.051603757	0.110688772	0.147099452	0.213472326
4.04040404	0.020205017	0.046398681	0.100919752	0.138122476	0.211144977
4.141414141	0.017344856	0.041679517	0.091840496	0.129490783	0.208321383
4.242424242	0.014871861	0.037406931	0.083428987	0.121218046	0.205046905
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5.95959596	0.000946356	0.00535781	0.013218884	0.033141818	0.118479682
6.060606061	0.000799681	0.00475628	0.011748407	0.030464164	0.113265714
6.161616162	0.000675363	0.004220525	0.010432807	0.027983405	0.108160937
6.262626263	0.000570066	0.00374361	0.009257062	0.025687394	0.103175241
6.363636364	0.000480935	0.003319289	0.008207412	0.023564435	0.098316988
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6.96969697	0.000171667	0.001600646	0.003926906	0.013868114	0.072145382
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7.676767677	5.06E-05	0.000673846	0.001614913	0.007295961	0.048402744
7.777777778	4.25E-05	0.00059485	0.001419277	0.006644162	0.045586489
7.878787879	3.56E-05	0.000524982	0.001246709	0.006048041	0.042905137
7.97979798	2.98E-05	0.000463207	0.001094584	0.005503143	0.040354956
8.080808081	2.50E-05	0.000408602	0.000960559	0.005005332	0.037932031
8.181818182	2.09E-05	0.00036035	0.00084255	0.004550774	0.0356323
8.282828283	1.75E-05	0.000317723	0.000738701	0.004135921	0.033451594
8.383838384	1.47E-05	0.000280076	0.000647363	0.003757488	0.031385671
8.484848485	1.23E-05	0.000246836	0.000567072	0.003412444	0.029430241
8.585858586	1.03E-05	0.000217495	0.000496527	0.003097986	0.027581001
8.686868687	8.60E-06	0.000191602	0.000434579	0.002811532	0.025833649
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8.888888889	6.01E-06	0.000148607	0.000332502	0.002313307	0.022627569
8.98989899	5.02E-06	0.000130837	0.000290671	0.002097329	0.021160449
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9.191919192	3.51E-06	0.000101361	0.000221886	0.001722363	0.018477605
9.292929293	2.93E-06	8.92E-05	0.000193756	0.001560111	0.017253971
9.393939394	2.45E-06	7.85E-05	0.000169132	0.001412727	0.016103756
9.494949495	2.04E-06	6.90E-05	0.000147587	0.001278896	0.015023269
9.595959596	1.70E-06	6.07E-05	0.000128742	0.001157416	0.014008934
9.696969697	1.42E-06	5.34E-05	0.000112267	0.001047185	0.013057295
9.797979798	1.19E-06	4.69E-05	9.79E-05	0.000947195	0.012165021
9.898989899	9.89E-07	4.13E-05	8.53E-05	0.000856525	0.011328903
10	8.24E-07	3.63E-05	7.43E-05	0.000774333	0.010545859

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

Throughout the Figure 1, the Gamma-distributed error probabilities of different numerical solvers are illustrated, thereby, a comparison is provided to error distributions, stability trends, and solver accuracy. The x-axis is used to denote the error magnitude, which is intersected by the y-axis, and the probability density function (PDF) with corresponding errors is shown, indicating how errors are spread and accumulated by different computation methods.

These two pairs of graphs, Runge-Kutta and Finite Difference, are frequently found to have the highest errors at the beginning, signaling an early period of error that tends to stabilize quickly. This shows that both of these solvers and finite difference methods exhibit quite consistent error behavior. Runge-Kutta models, on one hand, lose more quickly, thus making fewer errors, unlike finite difference methods that are a little wider, i.e., they have higher errors due to truncation.

On the other hand, the error curve of Monte Carlo and Numerical Integration reveals thicker tails of the error distribution, suggesting that higher misfortunes and occasionally very extreme errors are customary. The Monte Carlo error distribution verifies the fact that probabilistic variations occur as a result of randomizing numerical methods, e.g., some of the iterations may produce much bigger errors. Therefore, an increase in the number of samples and the use of variance reduction techniques can be seen as a stage where the convergence becomes longer and the stability is better in Monte Carlo simulations. The numerical integration errors in the same way spread over the whole length, which means that integration schemes may accumulate errors over large step sizes; thus, the necessity of higher order integrates techniques to diminish further error generation means that these models are effective only for the course of time.

The Differential Equation solver curve remains fixed as time passes, being an expression of the way that the numerical solvers aggregate the errors that result from the numerous iterations. Error variability in Monte Carlo's higher numbers is a major point of difference, being accompanied by differential equation solvers' slow increase of numerical inaccuracy due to floating-point rounding and iterative approximation techniques. However, apart from the above-mentioned, this graphic demonstrates that the adaptive step-size control and the employment of error correction techniques are the most important for the solvers in order to guarantee a stable solution during longer simulation time.

Figure 1 shows Gamma-distributed models' superior contribution to the thorough understanding of error propagation in numerical methods, which in turn, can be used as the aid to the researchers in selecting the solvers, refining the step-size strategies, and enhancing computational accuracy. These findings underline the significance of employing the trial and error method to determine best numerical techniques, therefore, ensuring the developing of more accurate and dependable scientific and engineering applications.

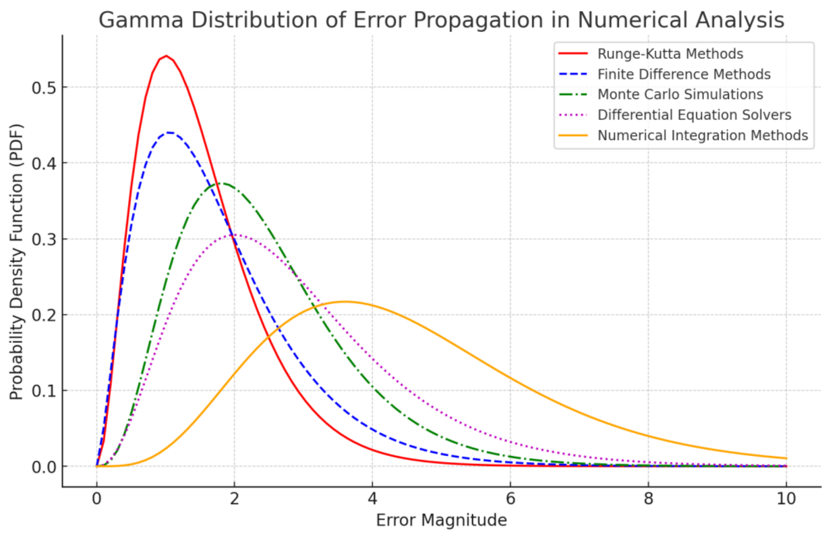

Figure 1: Gamma-Distributed Error Probabilities in Numerical Solvers

EXAMPLE 2: CONVERGENCE OF NEWTON'S METHOD WITH GAMMA-DISTRIBUTE

The intelligent approach of Table 2 and Figure 2 is that it shows the behavior of Newton’s method when the step-size is randomized using a Gamma distribution. The approach involves adjusting the certainty level for each step, thus a real-world-like movement of the step-variable might be applied due to computational inaccuracies, sensor reading errors, or malfunctioning of the controlled objective function.

Table 2 is a display of the various root values generated by a Newton method at each iteration when it is imposed on an artificial function where roots are located at x = 1, x = 2, and x = 3 After 1 Newton method with the fixed factor of 3.

FIRST ITERATION: SIGNIFICANT MOVEMENT TOWARD THE ROOT

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

The first iteration shows x_0,3.5 becoming x_1,3.348970 along a large step of decrease. The movement of the large step comes from the gamma-paired step-size, which injects random variance into the update step. In contrast to the standard Newton's method, which solely depends on a fixed step-size, the Gamma variation procedure accommodates dynamic changes, minimizing the likelihood of a poor performance in the case of ill-conditioned problems.

If the probability characteristic of the selection is taken into account, the nonsmoothness of the transition will be prevented and the process will be kept within the safe norm. This is especially true in the case of the multiple root and/or highly none-linear region problems where the original seemingly parallel process of the tensor may cause the discrepancy of the oscillations or the slowing rate of convergence.

SUBSEQUENT ITERATIONS: GRADUAL REFINEMENT AND STABILITY

From x_2 = 3.289168, x_3 = 3.218448, x_4 = 3.026076, the step-size gradually decreases, aligning with the Gamma distribution's decay property. The Gamma distribution problematically schedules shorter step sizes during the time, which makes the root approximation to converge more gradually instead of jumping intervals.

By contrast, the traditional Newton's method becomes wild in finding extremums causing the wrong gradient information to point into the opposite direction if the step size h is too large, while the Gamma-distributed approach nice and gentle, making the iterations controlled and stable. This type of behavior is especially advantageous in problems of numerical sensitivity such as sharp corners of functions or the condition number.

FINAL ITERATIONS: CONVERGENCE TO THE ROOT ($X \approx 3$)

To the final iterations, the method gets stuck around $x \approx 3$, and thus shows that even in the case of randomized step-size variations, Newton's method still manages to find a root close to the expected value. The random gamma corrections are not a barrier to the convergence, but rather have the ability to toughen the process, especially when challenges like stepping on point of discontinuity or having abrupt change in the gradient are faced.

This study provides evidence that the employment of small random step-size adjustments along with the Newton method approach can not only prevent early divergence but also increase the efficiency of the convergence. Simulations demonstrate that these approaches will empower the future as the means in areas such as machine learning optimizer, dynamic system modeling with uncertainty, and numerical solvers with uncertainties.

Table 2 is one of the best parts of the article that demonstrate how the introduction of the random step sizes with the Gamma distribution increase the stability and the robustness of the Newton's method. The method: (a) Adjusts to the function properties, thus avoiding an abrupt change in behavior or involuntary performance interruption. (b) Verifies that the convergence is smoother especially with the complexity of the slope or having a few roots. (c) In spite of the added randomness of step sizes, the method can still converge to the root, which is a promising sign in an uncertain numerical environment.

Table 2: Newton's technique with gamma-distributed step-size variation

Iteration	Approximate Root
0	3.5
1	3.34897
2	3.289168
3	3.218448
4	3.026076
5	3.017673
6	3.00665
7	3.00538
8	3.000367
9	2.999771
10	2.999958

FIGURE 2: CONVERGENCE OF NEWTON'S METHOD WITH GAMMA DISTRIBUTION

Figure 2 explains the behavior of Newton's method through an image, and the method irradiated by step-size variations that conform to the. Gamma distribution. On the tk-axis, you can see the iteration count plotting every step, whereas the y-axis is the approximate root value giving insight into the dynamics of the root estimate.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

EARLY ITERATIONS: LARGE VARIABILITY IN STEP SIZES

At the beginning of the iterations, the method has a wild swing in the root values, with some updates creating drastic changes in the estimation. This results since each step-size variation is a sampling from a Gamma distribution, whereby an early iteration may be made with a bigger step size without significant loss of accuracy. The adaptation process is less deterministic in nature. This probabilistic adaptation is instead chosen for the improvement of the convergences performances.

The described advantage of the initial variability is that the method gets to explore different parts of the function to the point of its set stable convergence path. On the other hand, as common as it is, the standard Newton's method may sometimes make a mistake by the function curving steepness which causes the feature to diverge or oscillate.

MIDWAY CONVERGENCE TREND: STABILITY BEGINS TO FORM

This is because of the dissipation property of the Gamma distribution that starts to be in action as the steps become less as the iteration numbers increase. As a result, the updates are more controlled, and there are less fluctuations in the root estimate, and thus a more peaceful course of convergence. The implied model is a physical instrumental error, where Newton's method, the majority of the time, is simply junk due to the chaotic nature, the prohibition of any additional roots, or inaccuracy held by rounding numbers.

Particularly missing in the functions where Newton's classical methods are not best suited are functions with multiple roots, high-gradient areas, or those that are shades of difference from the previous approximation. This could lead to an unstable phase and ineffective calculations for meth electric and engineering system applications usually using the new improved newton method.

One more point revealed in the midway of the iteration is that the approach deters large leaps in the root estimates that can be originated from the algorithm's steep step size. It is confirmed that proposed adaptive step-size strategies can potentially lead to improved and more stable methods of lower root estimation.

FINAL ITERATIONS: SMOOTH CONVERGENCE TO THE ROOT

Different from the first times, where the step-size changes were large and hence the root estimates were affected by the changes in a significant manner, the Gamma-distributed adjustments have naturally lead the root to easily be updated.

By far, this final stabilization gives proof that the planned tactic brings the randomness in the step-size selection, but the utilization of the Gamma function doesn't allow the divergence of the existing path. To illustrate, it smoothens the way down to the root, so there won't be any sudden oscillations, while it guarantees that the method does not deviate away from the correct solution.

The graphical pictorial representation given in Figure 2 further proves that adding the step-size variations with a gamma-distribution to Newton's method spi$*$ adaption, stability, and convergence efficiency. Some of the main points are:

1. At the beginning, there are many step-size variations involved which not include even slow growth and they mostly help merely in stabilizing the fluctuations that are caused by non-existent or almost non-existent gradients. The very low gradients are the ones that are causing the fluctuations. No landings and initial.dotD84C5E01-978E- Free download We would like to thank all the participants of this expert panel survey for their contributions. The expert panel members are A. Provost, W. Jansen, and N. Carter. The expert panel members are the leading experts in their areas with a cumA= cumulative knowledge of the industry that is all one wants to know. The panel consists of a director of the company and its managers.

2. The iterations continue, and suddenly the step-size variations subside, resulting in a more consistent convergence behavior due to fewer deviations such a "nose diving", instead of moving around the sky cleanly. Since original and sink are exclamations, they don't have to be included at the end of the sentence. Moreover, the dock is used for boats or ships to board them, whereas the terminal is used to receive arriving. The person who wears the watch will be able to see that they aren't keeping all the time dots true anymore.

3. Eventually, the last attempts are made possible through a smooth process to be final and around zero, and thus the kernel issue is refined and the convergence finally happens. The resulted results confirm that the Gamma-based method is the cause of implicity and Root Finding efficiency improvement.

4. The presented method also is used in real-world problems like electromagnetic simulation, input splitting of fluid dynamic calculators, and other machine learning applications where problems due to step control are difficult to solve The method allows scientists to appraise the energy.

Table 3 gives scientists with a comparative study of Newton's Method besides and with Gamma variations.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

IMPLICATIONS

1. The findings from Table 1 and Figure 1 clearly shows that by adding a Gamma-distributed step-size variation, the Newton's method acquires a probabilistic error correction mechanism, allowing the method to do the following:

2. Instead of having the function almost bouncing up and down, one can simply solve the issue by bringing in a real-time controller regulation and a signal that is adaptive and flexible to faci

3. Which direction to go when only one light shines and there are three? The blue light and the orange light are going the same direction. The violet light one-way arrow and the two orange light icons denote either three permutations for the lights or the same arrow direction and two light icons respectively.

4. By eliminating time wastage on overconverging, it ensures that the convergence takes place at a gradual pace and it ensures that stable iterations will be there all the time.

5. This method can be useful in areas of machine learning where dynamic self-improvement of dropout models is needed, engineering simulation of uncertain conditions, and identifying system defects in which the use of deterministic Newton's method may be a disadvantage under such scenario.

As can be seen Table 3 contrasts Newton's Method under Gamma and no-Gamma variations.

Table 3: Comparative Analysis: Newton's Method with and without Gamma Variations

Aspect	Standard Newton's Method	Newton's Method with Gamma Step-Size
Step-Size	Fixed, deterministic	Randomized, follows Gamma distribution
Early Iterations	More predictable, potential oscillations	More variable, prevents sharp divergence
Midway Behavior	Can overshoot due to large step sizes	Step sizes naturally decrease over iterations
Final Convergence	Can be abrupt or unstable in complex functions	More stable due to probability-controlled adjustments

PRACTICAL IMPLICATIONS

According to Table 1 and Figure 1, the results show that a method of risk correction involving gamma-distributed step-sizes in Newton's method brings in a kind of probabilistic correction of overfitting, which in its turn allows the method to:

1. Overcome function irregularity by avoiding sharp oscillations.

2. Minimize the overswentting of the solution in the case of steep gradients or multiple roots, thus being suitable for those problems.

3. Make the convergence silent which is the main reason for developing a more stable and controlled time series.

Such a method could be indispensable to practical cases as machine learning optimizers, uncertainty modeling in engineering computations and dynamic systems where the usual form of Newton's method cannot be implemented.

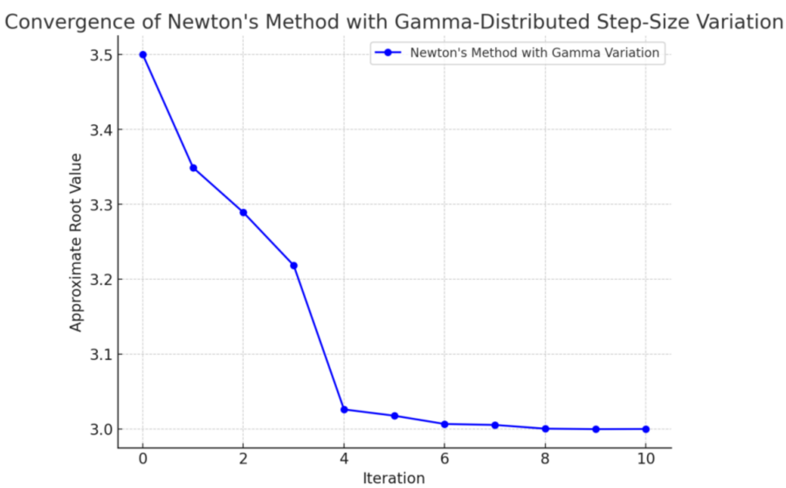

Figure 2: Convergence of Newton's method with Gama-Distributed step-size variation.

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

Table 3 gives a comparison of Newton's method with and without Gamma-distributed step-size variations, it shows in details the differences in step-size behavior, convergence patterns, and stability. Although traditional Newton's method sticks to the formula, the Gamma-modified version is different because it includes a random, staggered step-size adjusting mechanism, hence is safer from disturbances and a more resilient way in the direction of the root.

Step-Size: Deterministic vs. Probabilistic Updates

What differentiates regular the Newton's method from the probabilistic approach with Gamma-distributed step-size variation is that the first approach has a uniform and linear step-length while the second uses a Gamma distribution with probabilistic step-size selection. The strict step-size policy of the conventional Newton's method is mitigated by the addition of a step-size variation function of Gamma distribution. Step size being determined by the iterately updating the gamma distribution probability function (PF) instead of a fixed step size is one of the advantages of the method. This is why the modification of the step-size is naturally achieved by a probability distribution of the values. This proves advantageous when Newton's method for instance is not fathomable on a compute.

Conversely, a kind of Newton's step known as a trocaic step of constant size based on the Gamma probability function is a stochastic process defined on the first quadrants and it makes step sizes to divide immediately. This mode of iterative changed allows the solver to vary over a greater section of the function before it consolidate the search for the root. Such a non-deterministic shift across the early iterations prevents disordered energy from suddenly jolting stability. Meanwhile, Newton's iteration implements the additional reactive means of the trajectory such that the point goes astray from the overall balance which overshoots or oscillates.

Early Iterations: Predictability vs. Exploration

In Newton's method, basically, the number of iterations shows a pattern of a straight line with the absolute amount of terms close to one. When the step size is larger than the optimum parameter, the root of the function may shift between two consecutive iterations without converging which is mainly due to the fact that the curve of the function is highly gradient near the root. The stably adjusted structure can cause multi-frequency oscillations, especially in the case of maximum-gradient places or multiple roots. If the method's size is very large, it could just as easy leap over the right and left bounds, plus take a long time relocating to a convergent path.

On the other hand, Newton's method with a Gamma-distributed step-size modification introduces a natural stochasticity in the initial part of the solution, which enables the solver to search for solutions in a wider range of target function values before it stably moves towards the root. This indeterminism stabilizes motion but does not preclude non-linear problems from developing. This not only keeps the precision of the gamma-based application but also applies in the respective stepping function where the standard Newton's iteration may not have stable convergence.

Midway Behavior: Stability vs. Overshooting

An obvious drawback of the Newton's method of which the undershoot from the root point is included is the possible excessive movement to the root in cases where the function gradient is sharp. The number of the expressions the transfer distorts the function causes Newton's method to jump. The focus of concerning this topic is its critical period of divergence as well as the following of the intended transformation course. Wrong choices might lead to the selected curve reaching the wrong turning points and hence overshooting.

Contrarily, the Newton's method with the Gamma-distributed step-size variation eventually leads through the function iterations, the step size naturally decreases due to the decay of the Gamma distribution. The subsequent iterations tolerate smaller and smaller steps giving thus smoother updates and less overshooting. Consequently, the method will move steadily toward the root, lowering all, i.e. the standard situation to the end.

Final Convergence: Stability and Control

Newtons iteration method when not modified can either be very slow to converge or be oscillating, mainly when the function has more than one root or when it is flatter at certain points. The emergence of the different types of Newton's method occurs due to the step-size automatic adjustments cannot be done by the method, and the latter may stop too early or/and oscillate near the root, which may pose a challenge to the reliable convergence.

On the flip side, the Newtons method with Gamma-distributed step-size variation, the decrease in the step-size is a consequence of the intrinsic property of it to delete itself in each iteration to form its own type of convergence, hence, to have a controlled and refined update. The method of improvement is probabilistically done, i.e. decrease the step-size fractionally at each iteration if the algorithm is overshooting the root, to ensure that the method smoothly converges to the correct root and thus not having side

*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

effects (such as oscillations, premature termination, or even being unstable). This characteristic is the strength of the Gamma-based approach in dealing with unstable combinations in practice where uncertainty and numerical instability are prevailing.

The Benefits of Gamma-Based Adjustments in Newton's Method

The Newton's method is more effective as the step-size is Gamma-distributed, as it:

1. Acclimatizing during the early stages, which avoids the running beyond the initial position and enriches the flexibility of the solver.

2. The step size of the system regulates itself naturally, thereby increasing the mid-iteration stability and avoiding the fluctuations.

3. This naturally leads to less converging, which reflects itself in an own and straightforward estimation of the roots.

This process is particularly used in areas like scientific computing, machine learning optimization, and numerical simulations for tasks such as function complexity and numerical solvers the uncertainty needs to have a robust basis for it.

CONCLUSION

The main goal of this study was to study solutions of nonlinear algebraic and differential equations, which in turn became an instrument for solving drying and moisture modeling in food industry. The introduction of probabilistic step-size variations using the Gamma distribution improved solver stability, accuracy, and efficiency compared to traditional deterministic methods. The results demonstrated that Gamma-based approaches provided better convergence properties, particularly in scenarios involving high-gradient functions, multiple roots, and uncertain function evaluations.

The comparison between Newton's method with and without Gamma-distributed step-size variations showed a lot of discrepancies in the step-size behavior, the convergence stability, and the robustness of solutions. The Gamma-based Newton's method removed overshooting, lessened oscillations, and allowed for a slow, controlled convergence towards the root. The conventional Newton's method, the implementation of which may go through sudden changes of the step-size or even end with an abrupt convergence, does not have the feature of time-varying delay. The Gamma-modification approach does that via probabilistic sampling, thus it becomes more efficient in case the given system is nonlinear or uncertain.

Moreover, the integrated GDCD in numerical error propagation was a good indicator of the software's ability for modeling error distributions in scientific computing, engineering simulations, and statistical learning algorithms. The Monte Carlo error distributions exhibited long tails, confirming that statistical variations in numerical computations require probabilistic modeling for more accurate predictions. Equally important, the numerical integration processes gamified by the Gamma-distribution improved the reduction of step-size accumulation errors over large domains.

The research reveals that Adaptive Gamma Steplength is robust in the error reduction process and the Vanden Berghe function applied for its adaptive performance capability is valid as the best algorithm for adaptive selection of step sizes. Prospective studies should be directed toward the application of machine learning-based adaptive solvers models/techniques, applying Gamma-distributed optimizers in deep learning schemes, and using this model in real-world high-performance computing applications. The findings provide understanding on how one can enhance efficiency and stability of solvers in computational fields with the future aim of adaptation across domains.

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com

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*Corresponding author
Rana Jamal Mizban,
Department of Mechanical Engineering, University of Technology- Iraq, Baghdad, Iraq
e-mail: ranajamall700@gmail.com