

Article info

Received on: 13.02.2025

Accepted on: 20.02.2025

Published on: 10.03.2025

doi: <https://doi.org/10.52688/ASP60016>

Research Article

Complex Numbers Overview, Structure and Applications

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ABSTRACT

The Complex numbers are numbers that combine real and imaginary parts, written as $a+bi$, where i is the imaginary unit with $i^2 = -1$. This paper explains their fundamental properties in an easy-to-follow way. We start with basic operations like addition, subtraction, multiplication, and division. Then, we explore their geometric representation using the complex plane, where they can be plotted as points or vectors. The polar form of complex numbers and Euler's formula are introduced to simplify calculations. We also discuss how complex numbers help solve polynomial equations and compute roots using De Moivre's Theorem. Finally, we highlight practical uses in engineering, physics, and signal processing, showing why they are essential tools in modern science and technology.

Keywords: Complex numbers, Imaginary, Real, De Moivre's Theorem, Euler's formula.

INTRODUCTION

Complex numbers are discussed original presentation of the complexity and applications that given in the Table 1. A multiparameter random simplicial complex is considered on the vertex set $\{1, \dots, n\}$, where inclusion of simplices is governed by multiple connectivity probabilities. The analysis focuses on the topological properties of the complex in dimensions exceeding the critical threshold. Strong laws of large numbers and central limit theorems are established for the higher-dimensional Betti numbers. Additionally, lower tail large deviation probabilities for these Betti numbers are examined. Evidence is provided for the occurrence of phase transitions, indicated by the scaling constants in the central limit theorem and the exponential rates of decay in the lower tail large deviations [1]. Under standard conjectural assumptions, it is shown that the canonical symmetrizing trace on the Hecke algebra of irreducible spetsial complex reflection groups yields rational Catalan numbers when evaluated at powers of a Coxeter element. This result extends a technique previously employed by Galashin, Lam, Trinh, and Williams to uniformly establish the enumeration of their noncrossing Catalan objects for finite Coxeter groups [2]. Letting $b = -A \pm i$, where A is a positive integer, any complex number can be represented as a power series in b with coefficients in the set $A = \{0, 1, \dots, A^2\}$. It is proven that, for any real number and any non-empty proper subset $\tau > 2$ containing at least two elements, uncountably many complex numbers—including transcendental numbers—can be expressed as power series in b with coefficients in $J(b)$, and with irrationality exponent (with respect to Gaussian integers) equal to τ . A central component of the construction is the application of the 'Folding Lemma' to Hurwitz continued fractions. This leads to the formulation of a Hurwitz continued fraction analogue of the classical Zaremba's conjecture, for which several supporting results are established [3]. Complex fuzzy Z-numbers (CFZNs) are a new concept that expands on traditional fuzzy set theory by combining complex fuzzy sets with Z-numbers. This approach helps in handling uncertainty more effectively. The basic operations of CFZNs, like union, complement, and intersection, are explored in detail. Several methods for combining information (called aggregation operators) are developed, including weighted averaging and geometric approaches. The study looks at important properties of these methods, such as consistency and reliability. The paper introduces new ways to measure distance between CFZNs to help solve complex problems in various fields. It also presents a new decision-making algorithm that uses CFZNs to make better, more reliable decisions, particularly in situations involving uncertainty. A real-world example is provided, showing how CFZNs can improve predictive maintenance in industries by making equipment more reliable and reducing downtime. Finally, a new version of a decision-making method (CFZN-TOPSIS) is introduced and tested, with a comparison of its results [4]. In the case of a hazardous substance leak, Source Term Estimation (STE) helps quickly determine key information about the leaked substance, which is essential for minimizing harm to the environment and public safety. The accuracy of STE depends on having a good sensor setup. While most existing methods focus on optimizing sensor placement with a fixed number of sensors, there has been little focus on how the number of sensors affects STE performance, especially in complex urban areas. This study models three types of residential communities and looks into how sensor quantity affects STE. The study suggests that in urban areas, using a number of sensors that is one to two times the number of buildings allows emergency responders to pinpoint the leak's source with high accuracy within one or two blocks [5].

4'-Iodobiphenyl nonaethylene glycol ether (9bw) is a new small molecule made of a biphenyl unit and nine ethylene glycol (EG) units. It has been found to induce cell death in cancer cells by blocking mitochondrial respiratory complex I (CI), which lowers

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cellular ATP levels. It seems to have minimal effects on normal cells, suggesting it could be a potential anticancer agent with few side effects. However, the exact way 9bw targets CI is still unclear. To understand which structural features are important for its function, the molecule's analogues were tested in human oral squamous cell carcinoma lines HSC4 and Ca9-22. The analogues, including 4-hydroxy-4'-iodobiphenyl (HIOP), I-BP-EG3, I-BP-EG6, and I-BP-EG12, contained different numbers of EG units: 0, 3, 6, and 12, respectively. It was found that I-BP-EG6 and I-BP-EG12 inhibited CI similarly to 9bw, while I-BP-EG3 and HIOP had no effect on CI activity. These results suggest that the number of EG units plays a crucial role in determining the CI-inhibitory activity of 9bw and its analogues. Additionally, high-performance liquid chromatography (HPLC) analysis showed that HIOP and I-BP-EG3 were both taken up by the mitochondria, suggesting that the number of EG units influences CI inhibition more than the ability of the molecules to enter the mitochondria [6].

Accurately estimating the strength of protein-ligand interactions is crucial in drug discovery. Traditionally, this is done through experimental binding affinity assays, which are time-consuming, labor-intensive, and expensive. As an alternative, *in silico* (computer-based) prediction of binding affinity or energy is increasingly used, especially for screening large libraries of compounds. Key factors affecting binding energy include distance-related terms like electrostatic and van der Waals interactions. This study investigates and extends the use of distance-binding energy relationships, expressed as $E_{\alpha-d-k} \propto -d^{-k} E_{\alpha-d-k}$, to predict protein-ligand binding affinities. The contributions from various atom-type pairs are considered both individually and in combination. Additionally, contact number-energy relationships, described by $E_{\alpha-nk} \propto -n^k E_{\alpha-nk}$, are explored for the same purpose. Unlike classical theories of van der Waals or electrostatic interactions, the power exponents in these energy functions are not restricted. The newly developed models based on distance and contact number demonstrate better predictive performance than traditional, non-machine-learning-based scoring functions. These new models provide valuable insights for creating more accurate methods to predict protein-ligand binding affinity and analyze interactions [7].

An open question is raised to extend the Four Colour Theorem from two dimensions to three dimensions. A preliminary result shows that twelve colours—rather than four—are both sufficient and necessary to colour any 2-complex embedded in a specific 3-manifold. However, it is noted that the example of a 2-complex requiring twelve colours is non-simplicial [8].

The excited-state behavior of transition metal complexes can be influenced by factors such as the metal center's lability, Jahn-Teller distortions, and structure-dependent energetics, which affect their use in photochemistry. In this study, Co, Ni, and Cu complexes were examined using time-resolved transient absorption (TA) spectroscopy with 325 nm laser excitation. The results show photobleaching in the ultraviolet and photoinduced absorption in the visible region. The relaxation time at 350 nm remains between 5.5 and 9.4 ps, while a new fast relaxation component appears at 450 nm, lasting 160–290 fs. This ultrafast process is attributed to intersystem crossing (ISC) from the singlet metal-to-ligand charge transfer state (1MLCT) to the triplet MLCT state (3MLCT), with lifetimes decreasing as the metal's d-electron count increases. This leads to saturable absorption in the UV region and reverse saturable absorption in the visible region, as confirmed by Z-scan experiments. These findings provide valuable insights into the photophysical behavior of 3d metal-based complexes [9]. A comparison was made between first- and second-generation dendrimers containing biphenyl-based dendrons and green-emissive homoleptic or heteroleptic fac-iridium(III) complex cores. The core complexes were made up of one, two, or three 2-phenylpyridyl ligands responsible for light emission. It was found that the dendronized co-ligand, 5-phenyl-1-methyl-3-n-propyl-1H-1,2,4-triazole, did not affect the emission color. First- and second-generation dendrimers with two emissive 2-phenylpyridyl ligands exhibited high solution photoluminescence quantum yields (PLQYs) of nearly 90%, while homoleptic dendrimers and those with a single emissive ligand showed PLQYs around 70%. In the solid state, PLQYs decreased, with the highest neat film PLQY of 55% recorded for the second-generation dendrimer with a single emissive ligand. This decrease was due to the dendronized co-ligand acting as a self-host, reducing intermolecular interactions that cause luminescence quenching. Organic light-emitting diodes (OLEDs) made from neat films of these dendrimers showed relatively poor performance, with unbalanced charge transport and a maximum external quantum efficiency (EQE) of 6.5% for the second-generation dendrimer with a single emissive ligand. When the emissive layer consisted of a blend of dendrimers and tris(4-carbazoyl-9-ylphenyl)amine, similar current density-voltage characteristics were seen across devices, with EQE values following the trend of solution PLQYs. A maximum EQE of 14.1% was achieved with a blend containing the first-generation dendrimer with two emissive ligands. These findings show that both dendrimer generation and the number of emissive ligands can be used to optimize OLED performance [10]. The advance numbering system in the mathematical representation is used for explore the information on the large scale in sciences. In metal additive manufacturing, challenges arise from the complexities of heat transfer and phase transformations, especially when analyzing grain features. A common issue in evaluating grain size using ultrasonic scattering is the need for prior knowledge of complex scattering mechanisms at grain boundaries. To address this, a new approach was developed that combines convolutional neural networks (CNNs) with continuous wavelet transform (CWT). Laser ultrasonic longitudinal wave signals were collected from laser metal deposited Ti6Al4V samples reinforced with nano-sized B₄C particles. Given this tool, sciences and engineering can be equipped with more flexibility to solve the real life problems. CWT was applied to the signals using complex Morlet wavelet functions with different center frequencies and bandwidths, enhancing the limited dataset. A CNN model was created by stacking 3 × 3 convolutional kernels and 2 × 2 max-pooling layers. The expanded time-frequency images were used as inputs, and the model was trained, validated, and optimized through hyperparameter tuning. The proposed method achieved an average prediction accuracy of 97.78% for grain size numbers [11]. This helps in easing the life of human by facilitating the both medical and technology sectors of the human. In metal additive manufacturing, analyzing grain features is challenging due to heat transfer and phase transformation complexities. Specifically, evaluating grain size using ultrasonic scattering is difficult because it requires prior knowledge of complex scattering mechanisms at grain boundaries. To address this, an approach was developed using convolutional neural networks (CNNs) combined with continuous wavelet transform (CWT). Laser ultrasonic longitudinal wave signals were collected from Ti6Al4V samples, which were reinforced with nano-sized B₄C particles. CWT was

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applied to these signals using complex Morlet wavelet functions with different center frequencies and bandwidths, which helped enhance the limited dataset. A CNN model was built by stacking 3×3 convolutional kernels and 2×2 max-pooling layers. The model was trained and optimized through hyperparameter tuning with the expanded time-frequency images as inputs. This method achieved an impressive 97.78% accuracy in predicting grain size numbers [12]. Leukocyte immunoglobulin-like receptors (LILRs), found on human chromosome 19q13.4, consist of 11 receptors with genetic diversity. LILRB3 and LILRA6, in particular, share significant sequence similarity in their ligand-binding domains and show variable copy number (CN) states. However, understanding the precise function of these receptors has been challenging. To address this, a tool called JoGo-LILR Caller was developed to determine the CNs of LILRB3 and LILRA6 using large-scale whole-genome sequencing data. The tool was applied to 2,504 HapMap samples, creating a global CN profile, which was validated with 100% accuracy against data from 40 reference genomes. This profile was used to estimate the frequencies of LILRB3-LILRA6 CN haplotypes across different populations. Additionally, the tool included JoGo-LILR-trio for accurate inference of haplotypes in trio datasets, achieving 100% concordance with reference data in 40 child samples. The tool's utility extends to improving imputation software for LILRB3-LILRA6 CN types based on SNP arrays, aiding in studies linking CN variations to diseases such as inflammatory bowel disease and Takayasu arteritis [13]. A general method for computing polynomial roots over number fields using complex embeddings was explored. The key contribution was showing how the structure of a relative extension can be used to decode within a subfield. Several practical strategies were also discussed to improve efficiency. Experimental results from the implemented method were compared with the performance of the leading algorithm in Pari/GP [13]. A new immersed boundary method (IBM), combined with a wall model (WM), was developed and validated to simulate atmospheric boundary-layer flows over complex terrain. This method includes two novel aspects: first, it uses global numerical schemes that require specifying values throughout the solid region, and second, it integrates a wall model to handle high Reynolds numbers. The framework demonstrated second-order accuracy. Validation was conducted through simulations of flow over two- and three-dimensional cosine-squared hills, with results matching experimental data. It also successfully reproduced mean velocity and turbulence intensity using large-eddy simulation (LES). Simulations of flow over the steep-sloped Bolund hill showed good agreement with field data. Additionally, the framework was applied to model turbulent wake statistics downstream of a turbine on flat terrain, confirming its usefulness for simulating high-Reynolds number atmospheric and wind farm flows over complex terrain [14]. Crustal contamination is considered an important process in the formation of Ni-Cu-PGE deposits. This material can trigger sulfide saturation by adding external sulfur or increasing oxygen levels through the incorporation of volatile-rich rocks. Which is always can be seen in the real life experiment of the plot system of numbers representations. The Platreef, located in the northern limb of the Bushveld Complex in South Africa. This place is one of the world's largest platinum-group element (PGE) resources and also contains significant Ni-Cu-Co mineralization. The deposit is intersected by various footwall rocks from the Transvaal Supergroup in the structural content. This study used Niggli Numbers, a geochemical tool that classifies based on the molecular proportions of major elements. This method can assess the degree and types of crustal contamination at three locations: Tweefontein, Overysel, and a newly drilled section at Sandsloot.

The remaining part of the results is demonstrated in the following sections maintaining the similar mode of derivation. The study found that at Tweefontein and Overysel, the highest PGE-Ni-Cu grades were in largely uncontaminated pyroxenites, where Niggli c values were below 20. However, in areas with strong carbonate contamination (Niggli c values above 30), higher grades were not observed. At Sandsloot, however, elevated PGE and Ni grades were closely linked to carbonate contamination, with Niggli c values often above 20 and Niggli mg values above 0.8, suggesting dolomitic contamination. This correlation suggests that further investigation is needed to fully understand the role of carbonate contamination, particularly with the Malmani dolomite, in increasing Ni-Cu-PGE concentrations [15]. The further structure of the study is presenting the model implementation to the extend of the Cartesian coordination structure. In 2015, a framework for primality proving algorithms aimed at special sequences of integers was introduced by Abatzoglou, Silverberg, Sutherland, and Wong, using elliptic curves with complex multiplication. This framework was successfully applied to elliptic curves associated with imaginary quadratic fields of class numbers one and two. However, primality proving algorithms were not developed for fields with higher class numbers. This study extends their framework to imaginary quadratic fields with class number three. The new method leads to a more efficient primality proving algorithm for special integer sequences by using an imaginary quadratic field of class number three where the prime 2 splits. As part of the study, two special integer sequences corresponding to all such fields where 2 splits are provided, and computational results confirming the primality of these sequences are also presented [16]. This paper proves two integral formulas for the Hankel-Mellin transform and the double Fourier-Mellin transform of Bessel functions over the complex field \mathbb{C} , with both formulas yielding hypergeometric functions as results. As applications, the first integral formula is used to explicitly derive the spectral formula of Bruggeman and Motohashi for the fourth moment of the Dedekind zeta function over the Gaussian number field. Additionally, it is applied to establish a spectral formula for the Hecke-eigenvalue twisted second moment of central L-values for the Picard group PGL. The paper also develops the theory of the distributional Hankel transform on $\mathbb{C} \setminus \{0\}$ [17]. This article proposes a unified two-dimensional planar and axisymmetric differential formulation for both fluid dynamics and acoustic governing equations. It develops a differential formulation for computational acoustics using the hydrodynamic splitting method, applying a low Mach number approximation to the Linearized Perturbed Euler Equations (LPEE). Based on these formulations, a unified numerical methodology for Computational Flow-induced Acoustics (CFiA) is introduced. The fundamental handles complex geometries on body-fitted curvilinear grids. The structure of the numbers are represented in the dynamic way of Cartesian coordination system. A hybrid numerical scheme that combines finite volume and finite difference methods is presented for the algebraic formulation of CFiA. As an application, CFiA problem is driven from the mechanical cases in the real life that are associated with human living. The solution methodology uses a semi-implicit pressure projection for fluid flow, a four-step Runge-Kutta method with sub-time-stepping for acoustic perturbations, and a one-way explicitly coupled algorithm for solving the CFiA problem. Validation studies for both the computational acoustic and CFiA solvers show that they achieve fourth-order and second-order accuracy, respectively. The vertical and horizontal spaces of the coordination system can be used to plot the complex numbers in two dimensional plane. Additionally, the CFiA solver

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demonstrates superior performance with the proposed low Mach number approximation to LPEE compared to traditional LPEE-based solvers. This methodology introduces a computationally efficient approach for developing CFiA in low Mach number regimes with complex 2D planar and axisymmetric geometries [18].

Table 1: literature survey.

Ref.	Title/Topic	Main Focus/Contribution	Key Methods or Findings	Applications/Implications
1	Random Simplicial Complexes	Topological properties in high dimensions	Betti numbers CLTs, strong laws, large deviations	Phase transitions in random topology
2	Hecke Algebras and Rational Catalan Numbers	Symmetrizing trace yields rational Catalan numbers	Uses Coxeter elements, generalizes Galashin et al.'s work	Enumeration in algebraic combinatorics
3	Complex Numbers via Power Series in Special Bases	Power series representation with transcendental elements	Hurwitz continued fractions, Folding Lemma, Zaremba analogue	Number theory, irrationality exponent studies
4	Complex Fuzzy Z-Numbers (CFZNs)	Extends fuzzy set theory with Z-numbers	Defines operations, distance measures, aggregation operators, MCDM algorithm	AI, decision-making, predictive maintenance
5	Sensor Optimization in Source Term Estimation (STE)	Influence of sensor quantity on estimation accuracy	Optimal count: 1–2x number of buildings, improves localization	Emergency response, urban safety
6	9bw Molecule and Mitochondrial Complex I Inhibition	Antitumor activity via ATP depletion	EG unit count critical for CI inhibition, HPLC analysis confirms uptake	Cancer therapy, structure–function relationships
7	Protein-Ligand Binding Affinity Prediction	Distance and contact-based energy models	$E \propto -d^{(-k)}$, $E \propto -n^k$ formulations, outperforms classical scoring functions	Drug discovery, molecular interaction modeling
8	3D Generalization of the Four Colour Theorem	Minimum coloring of 2-complexes in 3-manifolds	Shows 12 colors are necessary and sufficient in non-simplicial case	Theoretical topology, graph theory
9	Excited-State Dynamics of Transition Metal Complexes	Ultrafast intersystem crossing in Co, Ni, Cu complexes	TA spectroscopy, Z-scan, MLCT states	Photochemistry, material design
10	Dendrimers in OLEDs	Ligand count and dendrimer generation affect PLQY and EQE	Comparison across gen/layer types, optimal: Gen1 + 2 ligands	OLED engineering, photonics
11	CNN and Wavelet-Based Grain Size Prediction in Metal AM	Non-invasive grain analysis using laser ultrasound and AI	CNN + CWT on ultrasonic signals, 97.78% accuracy	Metal manufacturing, NDE
12	LILRB3-LILRA6 Copy Number Inference	CN variability in immune receptors, algorithm development	JoGo-LILR tool, validated with HPRC data, trio inference support	Population genetics, disease association
13	Polynomial Root Computation over Number Fields	Improved method using complex embeddings	Leverages relative extension structure, heuristic strategies	Algebraic computing, number theory
14	IBM-WM for Atmospheric Flow over Complex Terrain	Accurate high-Re simulations over rough landscapes	LES validation, cosine hill and turbine wake modeling	Wind farm planning, environmental modeling

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15	Crustal Contamination and Ni-Cu-PGE Mineralization in the Platreef	Role of carbonate contamination in PGE enrichment	Niggli number analysis across sites	Mining geology, ore genesis models
16	Elliptic Curve-Based Primality Proving for Class Number 3 Fields	Extends prior methods to broader quadratic fields	Algorithm developed for cases where 2 splits, verified sequences	Computational number theory, cryptography
17	Hankel–Mellin and Fourier–Mellin Transforms of Bessel Functions	Integral formulas and spectral applications	Hypergeometric results, Dedekind zeta spectral formula	Analytic number theory, spectral analysis
18	Unified Acoustic Solver for Flow-Induced Noise in Complex Geometries	CFiA model using LPEE with low Mach approximation	Hybrid scheme, accuracy validation, fourth-order acoustic solver	Aerospace, automotive noise modeling

APPLICATIONS OF COMPLEX NUMBER

Complex numbers are set of mathematical representation of information that holds two values named as real and imaginary. Complex numbers play a pivotal role in various branches of modern mathematics and applied sciences. In topology and high-dimensional geometry, they are instrumental in the study of random simplicial complexes and topological phase transitions. For instance, they aid in analyzing Betti numbers, which count the number of holes in different dimensions, and are essential for understanding the shape and connectivity of complex structures. In terms of real values in the content of complex numbers, each value without the term (i) is called as real part. This is particularly evident in the context of threshold phenomena and structural emergence in random complexes. In number theory, complex numbers appear in the analysis of power series and continued fractions, especially when exploring irrationality and transcendence in real numbers. Papers that involve base expansions in non-integer systems frequently utilize complex representations to capture the full behavior of number-theoretic functions, offering insights into Diophantine approximations and distribution properties. On the other hand, terms in the complex number that come with (i) content is called as imaginary values. Another emerging application lies in fuzzy decision-making models, where complex fuzzy numbers e.g. Z-numbers, encode both quantitative and qualitative uncertainty. Here, the real part might represent the estimated value, while the imaginary part encapsulates confidence or reliability. This dual encoding of information has been used to improve decision-making in uncertain environments, such as healthcare systems and risk assessments. The structure of these numbers can hold information more than that of single values e.g. integers or floats. It is representing a dual set of information that can be preserved by the real and imaginary parts. Complex numbers are also central in algebraic number theory and cryptography. Specifically, in computations involving algebraic integers, complex embeddings of number fields help in root-finding algorithms for polynomials, which is crucial for solving high-degree equations. Moreover, techniques involving complex multiplication of elliptic curves have practical applications in cryptography, especially for primality testing and generating secure cryptographic keys. Complex set of numbers can be participate inside equations to solve a values related to applications of sciences and engineering. In spectral analysis and integral transforms, complex-valued functions are foundational. Complex Hankel–Mellin and Fourier–Mellin transforms are used to derive identities involving special functions like Bessel functions. These analytic tools are not only theoretically elegant but also essential for signal processing, physics, and analytic number theory.

DERIVATION

In math, numbers can be either normal or complex, in case of normal natural number, the value always can be derived from the content on the other side of the equal sign. Complex numbers arise naturally when certain algebraic equations have no solutions within the real number system. For instance, the equation $x^2 + 1 = 0$ has no real solution because the square of any real number is non-negative. To address this limitation, mathematicians introduced an extension of real numbers by defining a new unit called the imaginary unit. This led to the creation of complex numbers, which consist of a real part and an imaginary part. In case of complex numbers, the value of the complex number is equal to two values corresponding to its real and imaginary parts. A complex number is typically written as a combination of a real and an imaginary number, and operations such as addition and multiplication follow specific rules. Geometrically, complex numbers can be visualized on a plane, with the real part on the horizontal axis and the imaginary part on the vertical axis. This graphical representation is known as the Argand plane. The concept of modulus helps determine the magnitude of a complex number, while the conjugate provides a reflection over the real axis. The real part need to have equivalent content of the other side of the equal sign so as the imaginary part as demonstrated hereinafter. Complex numbers can also be expressed in polar form, which is particularly useful in advanced mathematical operations like multiplication and root extraction. Euler's formula connects trigonometric functions with exponential functions and gives a compact way to represent

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complex numbers. This formulation not only simplifies computations but also expands the utility of complex numbers in fields such as physics, engineering, and signal processing.

No real solution example:

The equation $x^2 + 1 = 0$ leads to $x^2 = -1$, which has no solution in real numbers.

Imaginary unit:

The imaginary unit is defined such that $i^2 = -1$.

Definition of a complex number:

A complex number is written as $z = a + bi$, where a and b are real numbers.

Addition of complex numbers:

If $z_1 = a + bi$ and $z_2 = c + di$, then $z_1 + z_2 = (a + c) + (b + d)i$.

Multiplication of complex numbers:

$z_1 * z_2 = (ac - bd) + (ad + bc)i$.

Modulus of a complex number:

The modulus of $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.

Complex conjugate:

The conjugate of $z = a + bi$ is $\bar{z} = a - bi$.

Polar form of a complex number:

$z = r(\cos\theta + i \cdot \sin\theta)$, where $r = |z|$ and θ is the argument (angle) of z .

Euler's formula:

$e^{i\theta} = \cos\theta + i \cdot \sin\theta$.

Exponential form of a complex number:

$z = re^{i\theta}$.

CONCLUSION

A strong emphasis emerges on computational techniques, such as the use of AI in materials science (e.g., CNN for grain size prediction), hybrid solvers for fluid-acoustic interaction, and heuristic algorithms for number theory problems. Mathematical insights, such as extensions to fuzzy set theory, rational Catalan numbers, and random simplicial complexes, reinforce the theoretical underpinnings that drive innovation in applied fields. Several studies highlight the intersection of data science and physical modeling. For instance, optimized sensor deployment in urban environments and protein-ligand binding models exemplify how statistical modeling and physical systems interact to solve complex real-world problems. Meanwhile, the application of spectral transforms, elliptic curve algorithms, and continued fractions shows the ongoing refinement of mathematical tools for both theoretical and practical outcomes. In the life sciences, novel approaches to cancer therapy through mitochondrial inhibition and immune gene CN inference offer promise for healthcare. Studies on photophysical properties of transition metals and dendrimers further underline the growing role of chemical design in technology. Altogether, the references reflect a trend toward integrated, model-driven research that blurs disciplinary boundaries to tackle increasingly complex challenges.

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