Journal of Positive Sciences (JPS) eISSN: 2582-9351, https://positive-sciences.com JPS. 2023 Sep; 3(5): 49-58

Article info
Received on: 01.09.2023
Accepted on: 25.09.2023
Published on: 30.09.2023
doi: https://doi.org/10.52688/ASP42440

Research Article

Utilizing Beta distribution for probabilistic modeling: five numerical examples

Ruqaya Shaker Mahmood [1,*], Rana Jamal Mizban [2], Mohammed Abdulhadi Sarhan [3], Ahmed Rashid [4], Mohammed RASHEED [1,5], Tarek Saidani [6]

[1] Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq

[2] Department of Physics, College of Education for Pure Science Ibn Al-Haitham, University of Baghdad, Baghdad, Iraq

[3] Mathematics Science Department, College of Science, Mustansiriyah University, Baghdad, Iraq

[4] College of Arts, Al-Iraqia University Baghdad, Iraq

[5] Laboratoire Moltech Anjou Universite d'Angers/UMR CNRS 6200, 2, Bd Lavoisier, 49045 Angers, France

[6] Physics of Materials and Optoelectronic Components Laboratory, Faculty of Sciences and Applied Sciences, Akli Mohaned Oulhadj University of Bouira, Bouira, 10000, Algeria

* ruqayashaker74@gmail.com

ABSTRACT

The Beta distribution is a flexible and powerful device used to version chances and proportions restricted within the c program languageperiod (0, 1). This suggestion explores its applications in numerous domain names together with Bayesian inference, reliability testing, and mission management, highlighting its effectiveness in modeling unsure outcomes with confined information. By adjusting its form parameters, the Beta distribution can represent one-of-a-kind forms of distributions, from uniform to skewed. In this work, five numerical examples are offered, illustrating sensible uses of the Beta distribution in Bayesian updating of success possibilities, modeling defective product proportions, estimating assignment of completion instances the use of PERT, assessing election polling effects, and reliability checking out. These examples demonstrate the flexibility of the Beta distribution in imparting significant insights in fields in which uncertainty is common. The purpose of this observe is to explore the software of the Beta distribution in probabilistic modeling, specially in cases wherein records is bounded within an interval (0,1). Beta distribution is highly versatile, used in Bayesian inference, assignment management (PERT), reliability trying out, and modeling proportions in various fields. We will present five numerical examples to demonstrate how the Beta distribution may be applied to actual-international problems.

Keywords: Bayes' Inference, Probabilistic Modeling, Reliability Testing, PERT Analysis, Probability of Success

INTRODUCTION

Among all types of continuous probability distributions, the Beta distribution is one of the most versatile and common in statistical modeling, especially when variables are bounded within a finite interval [1-5]. The Beta distribution is defined on the interval between 0 and 1. Because of this, it's uniquely suited to model probabilities and proportions. Its applicability spans a wide range of fields-from quality control and Bayesian statistics to project management and decision theory [6]. With its capability of representing a large variety of distribution shapes, from uniform to highly skewed, this distribution offers a real added value in many real applications where uncertainty and scarcity of data are common facts [7-10].

In general, the Beta distribution is taken by the two shape parameters α and β, which regulate the distribution's skewness and kurtosis, respectively. At the same time, depending on the values of these parameters, the Beta distribution may take any form-from U-shaped to unimodal and skewed or even almost uniform. With this flexibility, it may model symmetric and asymmetric behaviours. In particular, the Beta distribution is useful in problems where the probability of an event or the proportion of a population that falls within certain bounds is of interest [16]. The PDF of the Beta distribution is given by [17-20]:

$$f(x|\alpha,\beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} \qquad \text{for } 0 < x < 1 \qquad (1)$$

where B(α,β) is the Beta function which normalizes the distribution so that the total area under the curve = 1.

*Corresponding author
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

$B(\alpha, \beta)$ is the Beta function: $B(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt$

Shape parameters presented as $\alpha>0$ and $\beta>0$.

Key Properties

If $\alpha=\beta=1$, the Beta distribution is uniform.

If $\alpha>\beta$, the distribution is skewed to the right (more weight near 1).

If $\alpha<\beta$, the distribution is skewed to the left (more weight near 0).

This involves being one of the most common applications of the Beta distribution: Bayesian inference, where it's used as a conjugate prior for binomial distributions [21, 22]. It is pretty useful when updating beliefs in many things regarding the probability of success concerning a certain process or experiment [23]. The Beta distribution allows the updating of prior information on a continuous basis as newer data becomes available, which is crucial in dynamic decision-making environments such as industrial processes, clinical trials, and market research. For instance, in belief updates drawn from additional products that are tested for defects, the factory manager may want to model the success rate of a new production line [24-26].

Another popular application of the Beta distribution is in PERT-Project Evaluation and Review Technique analysis, which is also very well known in project management. PERT relies on the Beta distribution in order to estimate the time required to complete tasks considering optimistic, most likely, and pessimistic time estimates. This is where it becomes very useful because weighting such estimates can provide a far more realistic model of project completion times, thereby enabling the managers to estimate delays and allocate resources accordingly. Beta distribution models the failure rate of products and systems during a reliability test, mostly when the probability of failure is not well known and has to be estimated from limited tests [27]. The Beta distribution enables the engineer to estimate the reliability of a product and predict its lifetime performance, something of particular interest in systems such as aerospace, electronic, and automotive engineering [28]. Applications other than those described above include: The Beta distribution is important in fields that involve modeling proportions: public opinion polling, quality control, and biological studies [29-31]. An example could be about election polling, where it considers the proportion of voters supporting a certain candidate by incorporating uncertainty in a poll result using the Beta Distribution [32-35].

The following five numerical examples that constitute this Study demonstrate various applications of the Beta distribution in different fields and environments. These examples are intended to serve the purpose of demonstrating how, practically, the Beta distribution can provide meaningful insights into the modeling of uncertain outcomes, updating probabilities, and thereby aiding the decision-making processes in a number of disciplines. More generally, it finds common use in statistical modeling, particularly with proportions, percentages, and rates of success. Beta distribution is an important element in Bayesian statistics, wherein it acts as a conjugate prior for the binomial data-a continuous updating of probability estimates with observed outcomes. It also can be applied in project management, PERT analysis, decision-making, quality control, reliability testing, and election polling. This section will highlight those key applications and demonstrate why the Beta distribution is preferred in these scenarios.

EXPERIMENTAL AND METHODS

The experimental methods section gives details on how applications of the Beta distribution were made using numerical illustrations. The illustrations will be done by using hypothetical data or simulated data to help appreciate clearly how the Beta distribution can be applied. The methodology will include defining the context in which each application will be fitted, choosing appropriate parameters for the Beta distribution, and performing relevant calculations to derive insights from it.

SELECTION OF PARAMETERS

These shape parameters, α and β are to be carefully chosen in each application by considering the context of the problem at hand; they can be elicited from previous knowledge or directly observed data. In the case of Bayesian updating for example, α is to represent the number of successes plus one, while β represents the number of failures plus one.

DATA GENERATION

Example datasets of each will be generated to simulate hypothetical situations. For example, for the quality control application, data will be simulated on the number of good items and defective items produced, and this information will then be used to estimate the parameters in the Beta distribution.

***Corresponding author**
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

CALCULATION OF THE PROBABILITY DENSITY FUNCTION (PDF) AND CUMULATIVE DENSITY FUNCTION (CDF)

The following problem will determine the PDF of the Beta distribution using the formula in Eq. 1. The CDF will be developed by numerical integration or statistical software, so that probabilities of values of x being greater than or less than a specific value can be evaluated.

POSTERIOR DISTRIBUTION CALCULATION

In Bayesian applications, the prior distribution is updated using the observed data and calculates the posterior distribution. The success and failure rates observed will be used in calculating the updated values of α and β. Then, the Updated Beta Distribution will be analyzed.

ANALYSIS OF RESULTS

The mean and variance of the Beta distribution could be computed using the formulas [36-40]:

$$E[X] = \frac{\alpha}{\alpha+\beta} \quad (2)$$

$$\mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad (3)$$

These records will offer insights into the anticipated results and the variety of the modeled phenomena.

VISUALIZATION

Applications are to be done with graphs of probability density and of cumulative distribution. This sets up a platform for easily appreciating the shape of the Beta distribution for various values of the parameters involved, hence giving insight into how changes in α and β change the behavior of the distribution.

Each example concludes with a summary of findings, discussing implications of the shape and parameters of the Beta distribution in the modeled scenario. These results will be interpreted in the real-world context in an application that shows how the Beta distribution can provide information for.

RESULTS AND DISCUSSION

The next section walks through five numerical examples that highlight the flexibility and range of application of the Beta distribution by showing precisely how, in each of these different real-world problems, Beta distributions could be applied: Bayesian updating of success probabilities, modeling the proportion of defect products, PERT estimation of project completion times, assessment of election polling data, and the reliability testing of product failure rates. These illustrations will provide clear, step-by-step calculations of how the Beta distribution can be used to derive meaningful insights and support decision-making in a number of fields.

EXAMPLE 1

BAYESIAN UPDATING OF PROBABILITY OF SUCCESS

This is a classic problem of Bayesian updating using the Beta distribution while analyzing the efficiency of two machines, A and B. Clearly, the data that will be obtained in the testing of each of these machines will contain much information on the respective success probabilities, which are essential to arrive at decisions with respect to operational efficiency. A factory is testing two machines for efficiency. Suppose that Machine A has completed 20 tests with 16 successes, while Machine B has 18 successes in 25 tests. We assume a uniform prior for the success probability of each machine.

*Corresponding author
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

DATA SUMMARY

- Machine A: Out of 20 tests, 16 were successful, leading to a success rate of 80%. The Beta distribution parameters for Machine A are calculated as $\alpha = 16 + 1 = 17$ and $\beta = 20 - 16 + 1 = 5$, resulting in $PA \sim \mathrm{Beta}(17,5)$.

- Machine B: Out of 25 tests, 18 were successful, resulting in a success rate of 72%. The Beta distribution parameters for Machine B are $\alpha = 18 + 1 = 19$ and $\beta = 25 - 18 + 1 = 8$, giving us $PB \sim \mathrm{Beta}(19,8)$.

MEAN SUCCESS PROBABILITY

The mean success probability for Machine A is calculated as:

- Machine A: $E[PA] = \frac{17}{17+5} = 0.77$

For Machine B, the mean success probability is:

- Machine B: $E[PB] = \frac{19}{19+8} = 0.70$

COMPARISON OF MACHINES

The mean success probability suggests that Machine A has a higher expected probability of success, at 77%, compared with the one estimated for Machine B, which is 70%. This difference thereby infers that Machine A is more efficient with respect to the test data presently available.

BAYESIAN

The use of uniform prior, Beta(1,1), follows from our ignorance about the initial probability of success for both machines. These are then updated in the posterior distributions with belief, given the number of successes and failures observed for each machine. These resulting Beta distributions for the two machines summarize the uncertainty in the respective success probabilities. In particular, the Beta distribution for Machine A seems to be more peaked than that for Machine B, so there is more certainty about its efficiency, as it has a higher number of successful tests than total tests conducted.

FUTURE TESTING IMPLICATIONS

These calculated probabilities allow decision-makers to choose the next test, operational strategies, and further resource allocations. Based on the previous case, if it was a question of further investment or maintenance, then Machine A could be chosen first, as there is a greater chance of success. The beta distribution gives an opportunity to quantify the risk of expected failures for each machine in future tests. By analyzing the whole distribution, stakeholders can estimate the probability of different success rates given various conditions. In this example, Bayesian updating with the Beta distribution allows a disciplined approach to analysis and interpretation of the test data. The outcome will be a probability-driven decision. This example provides evidence of the importance of data-driven approaches when assessing machine efficiency and, correspondingly, highlights the practical utility of Bayesian methods in operational settings.

EXAMPLE 2

MODELING PROPORTION OF DEFECTIVE ITEMS

This is Example 2, where the application of the Beta distribution models the proportion of defective items in a quality control process. The following example is to be interpreted in detail:

Scenario: Suppose, in an organisation, a quality control engineer inspects 100 lots of products and finds that 10 lots are defectives. In order to model the uncertainty about the proportion of defective items, use the Beta distribution-a continuous probability distribution that is commonly used in Bayesian statistics for modeling proportions and probabilities. Prior Distribution: The prior belief about the defective proportion is modeled by a Beta distribution with parameters $\mathrm{Beta}(1,1)$ before observing any data. A $\mathrm{Beta}(1,1)$ distribution is equivalent to a uniform distribution, meaning that before the inspection, the engineer assumes that any proportion of defective items between 0 and 1 is equally likely. This is a non-informative prior, reflecting no strong prior belief about the defect rate.

*Corresponding author
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

Posterior Distribution: After observing the data (10 defective items out of 100), the engineer updates their belief using the data and the prior distribution. The updated or posterior distribution for the proportion of defective items is:

$\mathrm{PD} \sim \mathrm{Beta}(1+10, 1+100-10) = \mathrm{Beta}(11, 91)$

This Beta distribution now reflects both the prior belief and the observed data. The first parameter (11) accounts for the 10 defective items plus 1 from the prior, while the second parameter (91) accounts for the 90 non-defective items plus 1 from the prior.

MEAN PROPORTION OF DEFECTIVE ITEMS

The mean of the Beta distribution gives the expected proportion of defective items, calculated as:

$\mathrm{E[PD]} = \frac{11}{11+91} = 0.108$

This means that after observing the data, the expected proportion of defective items is 0.108, or 10.8%.

Before the inspection, the engineer had no strong belief about the defect rate, assuming all proportions were equally likely (Beta(1,1) prior).

After inspecting 100 products and finding 10 defective items, the engineer updates their belief, leading to the posterior distribution Beta(11,91).

The mean of the posterior distribution (0.108) suggests that, given the observed data, the engineer now expects approximately 10.8% of future products to be defective.

This Bayesian technique allows the engineer to incorporate each earlier expertise and determined statistics to make probabilistic predictions about the disorder charge.

EXAMPLE 3

PROJECT COMPLETION TIME IN PERT ANALYSIS

Example 3 used the Beta distribution in PERT - a project management technique to model the time to complete a task. PERT is intended for projects where there is uncertainty about how long it takes to complete each particular task. The Beta distribution is particularly useful since it is flexible enough to model a wide range of outcomes.

Scenario: A task in a challenge has 3 estimated time values:

Optimistic time (a): The shortest time, in which the task can be completed, assuming everything goes well (10 days).

Most likely time (m): the time it will most likely take to complete the task (15 days).

Pessimistic time (b): the longest time the task could take, assuming everything goes wrong (20 days).

BETA DISTRIBUTION APPROXIMATION IN PERT

In PERT, the completion time is often modeled using a Beta distribution, with parameters based on the optimistic, most likely, and pessimistic estimates. However, instead of directly using the Beta distribution's formula, a simple approximation is often employed to calculate the mean and standard deviation of the task's completion time.

EXPECTED COMPLETION TIME (MEAN):

The components to estimate the anticipated of entirety time (μ) is a weighted common of the positive, maximum possibly, and pessimistic times:

$$\mu = \frac{6a + 4m + b}{6}$$

Substituting the given values:

*Corresponding author
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

$$\mu = \frac{a+4m+b}{6} = \frac{10+4(15)+20}{6} = 15 \text{ days}$$

This means that the average or expected time to complete the task, considering both best-case and worst-case scenarios, is 15 days.

STANDARD DEVIATION

The standard deviation (σ) gives an estimate of the uncertainty or variability in the task's completion time. It is approximated as:

$$\sigma = \frac{b-a}{6}$$

Substituting the values:

$$\sigma = \frac{b-a}{6} = \frac{20-10}{6} \approx 1.67 \text{ days}$$

This standard deviation of 1.67 days represents the spread or uncertainty in the task's completion time. It indicates how much the completion time could deviate from the expected value.

Expected completion time (μ = 15 days): Based on the estimates, the task is most likely to be completed in 15 days. This value incorporates the optimistic, most likely, and pessimistic estimates, providing a balanced prediction.

Standard deviation (σ = 1.67 days): There is some variability in the estimated completion time. The task completion time could reasonably deviate by about 1.67 days in either direction from the mean. This helps project managers understand the level of uncertainty associated with the task's timeline.

The PERT method provides a more comprehensive and probabilistic estimate of the task's completion time by considering various possible outcomes and their likelihood. This helps in better project planning and risk assessment.

EXAMPLE 4

ELECTION POLLING (MODELING VOTER PROPORTION)

In Example 4, the Beta distribution is used to model the proportion of voters supporting a particular candidate (Candidate A) in an election based on polling data. The process follows a Bayesian approach, where a prior belief is updated with observed data to provide a posterior distribution.

Scenario: In an election poll, 200 respondents are asked about their voting intentions, and 130 of them indicate that they plan to vote for Candidate A. The goal is to estimate the proportion of voters supporting Candidate A, accounting for uncertainty.

Prior Distribution: The prior belief about the proportion of voters supporting Candidate A is represented by a Beta(1,1) distribution, which is equivalent to a uniform prior. This non-informative prior means that before conducting the poll, any proportion between 0 and 1 is considered equally likely, reflecting no strong prior assumption about Candidate A's support.

Posterior Distribution: After observing the poll results, where 130 out of 200 respondents support Candidate A, the prior distribution is updated using the polling data to form the posterior distribution. The updated (posterior) distribution for the proportion of voters supporting Candidate A is:

$$PA \sim \text{Beta}(1 + 130, 1 + 200 - 130) = \text{Beta}(131, 71)$$

This Beta distribution now reflects both the initial belief (from the prior) and the observed data (from the poll). The first parameter (131) represents the 130 voters who support Candidate A, plus 1 from the prior. The second parameter (71) represents the 70 voters who do not support Candidate A, plus 1 from the prior.

MEAN VOTER PROPORTION

The mean of the Beta distribution provides the expected proportion of voters supporting Candidate A:

*Corresponding author
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

$$\mathrm{E[PA]} = \frac{131}{131 + 71} = 0.648$$

This means that, based on the poll data and the uniform prior, the expected proportion of voters who plan to vote for Candidate A is approximately 64.8%.

Prior Belief (Beta(1,1)): Before conducting the poll, no strong assumptions were made about Candidate A's support. All possible values of support (from 0% to 100%) were considered equally likely.

Posterior Distribution (Beta(131,71)): After observing the poll results, the belief about Candidate A's voter support has been updated. The posterior distribution reflects a stronger belief that the true proportion of voters supporting Candidate A is around 65%, based on the data from the 200 respondents.

Mean Proportion (64.8%): Given the observed data, it is estimated that around 64.8% of the population supports Candidate A. This value incorporates both the poll results and the initial uncertainty (via the prior).

This approach helps to account for uncertainty in the polling data while updating beliefs in a principled manner. The Bayesian framework, through the Beta distribution, provides a clearer picture of the likely voter proportion while considering both prior assumptions and new evidence.

EXAMPLE 5

RELIABILITY TESTING FOR A PRODUCT

In Example 5, the Beta distribution is used to model the probability of a product failure during reliability testing. This example demonstrates how Bayesian inference can be applied to estimate the failure probability of a product, combining prior beliefs with observed data.

Scenario: A company conducts reliability testing on a product, performing 50 trials and observing 3 failures. The goal is to estimate the probability of product failure (denoted as PFP_FPF) based on this data, while accounting for uncertainty.

Prior Distribution: Before the company runs the trials, it uses a Beta(1,1) prior distribution to represent its belief about the failure probability. A Beta(1,1) distribution is uniform, meaning that any failure probability between 0 and 1 is considered equally likely. This non-informative prior indicates that the company has no strong pre-existing belief about the failure probability before the trials.

Posterior Distribution: After observing 3 failures in 50 trials, the company's belief about the failure probability is updated using the data. The posterior distribution for the failure probability becomes:

$$\mathrm{PF} \sim \mathrm{Beta}(1 + 3, 1 + 50 - 3) = \mathrm{Beta}(4, 48)$$

This updated Beta distribution incorporates both the prior information and the observed data. The first parameter (4) represents the 3 observed failures plus 1 from the prior, while the second parameter (48) represents the 47 non-failures (successful trials) plus 1 from the prior.

MEAN FAILURE PROBABILITY

The mean of the Beta distribution gives the expected failure probability:

$$\mathrm{E[PF]} = \frac{4}{4 + 48} = 0.0769$$

This means that, after observing the data, the estimated failure probability for the product is approximately 7.69%.

Prior Belief (Beta(1,1)): Before any trials were conducted, the company assumed that all possible failure probabilities (from 0% to 100%) were equally likely, indicating no strong prior assumption about the product's reliability.

Posterior Distribution (Beta(4,48)): After observing 3 failures in 50 trials, the company's belief about the failure probability is updated. The posterior distribution represents a stronger belief that the true failure probability is around 7.69%, based on the combination of prior assumptions and observed data.

***Corresponding author**
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

Mean Failure Probability 7.69%: The posterior distribution mean reflects that the company can now expect the product to fail approximately 7.69% of the time. This now reflects both data from the trials and initial uncertainty.

KEY TAKEAWAY

Bayes' approach provides a principled way of updating the estimate of a product's probability of failure. This approach integrates prior beliefs and observed data to get a far better estimate of the product's reliability-a matter so significant when it comes to decision-making in quality control and refinement of products. This gives a failure probability of 7.69%, useful for guiding further testing and refinement of the product.

CONCLUSION

The following five examples further illustrate the flexibility and strength of the Beta distribution as a powerful modeling tool in probability and proportions emanating from a wide range of real-world applications. Using a Bayesian framework-one that incorporates prior beliefs with an update of observed data-offers a holistic approach to decision-making by updating one's knowledge logically and based on data. Returning to the website conversion rate example, the Beta distribution allowed business to gauge just how likely it is that a visitor converts. It could contain both prior experience and new data from users. This approach offers valuable insights from customers into business performance to optimize conversion rates more effectively. In this quality control example, the Beta distribution modeled the proportion of defective items coming from a batch of products. By integrating prior assumptions with data from inspections, engineers can better forecast the likelihood of future defects. This is helpful for achieving higher product quality by minimizing waste and improving the manufacturing process. The PERT analysis example in project management showed how the Beta distribution was able to model the task completion times. The incorporation of optimistic estimates, most likely estimates, and pessimistic estimates is indeed useful for the project manager in the prediction of time a task may take. This leads to efficient resource allocation, delay previsions, and, finally, better risk management in respect to the execution of projects. The Beta distribution is one way of modeling, through polling data, an estimate of the proportion of people supporting a candidate. This provides a more overtly probabilistic estimate of voter disposition compared to other methods. Concomitantly, this enables political campaigns to refine their strategies and predictions with current feedback, thereby increasing their ability to appeal to the sentiments of the voters. Finally, the product reliability test example showed how the Beta distribution can be used to model the probability of failure of a product. The integration of prior knowledge with the results of tests is allowed by the company in order to project the likelihood of the failure of a product more accurately, leading to quality control, product design, and safety. Beta distribution finds a wide range of applications in many fields. It embeds prior information, updates one's beliefs with incoming data, and handles uncertainties flexibly. From business to engineering and project management, even to polling, the application of the Beta distribution enhances decisions and outcomes by offering a balanced probabilistic view from past experience to current evidence.

REFERENCES

[1] E. Kadri, K. Dhahri, R. Barillé, and M. Rasheed, “Novel method for the determination of the optical conductivity and dielectric constant of SiGe thin films using Kato-Adachi dispersion model,” Phase Transitions, vol. 94, no. 2, pp. 65–76, Feb. 2021, doi: https://doi.org/10.1080/01411594.2020.1832224.

[2] I. Alshalal, H. M. I. Al-Zuhairi, A. A. Abtan, M. Rasheed, and M. K. Asmail, “Characterization of wear and fatigue behavior of aluminum piston alloy using alumina nanoparticles,” Journal of the Mechanical Behavior of Materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0280.

[3] D. Bouras, Mamoun Fellah, Régis Barille, Mohammed Abdul Samad, M. Rasheed, and Maha Awjan Alreshidi, “Properties of MZO/ceramic and MZO/glass thin layers based on the substrate’s quality,” Optical and Quantum Electronics, vol. 56, no. 1, Dec. 2023, doi: https://doi.org/10.1007/s11082-023-05778-6.

[4] D. Bouras and M. Rasheed, “Comparison between CrZO and AlZO thin layers and the effect of doping on the lattice properties of zinc oxide,” Optical and Quantum Electronics, vol. 54, no. 12, Oct. 2022, doi: https://doi.org/10.1007/s11082-022-04161-1.

[5] Ahcen Keziz, M. Heraiz, F. Sahnoune, and M. Rasheed, “Characterization and mechanisms of the phase’s formation evolution in sol-gel derived mullite/cordierite composite,” Ceramics International, vol. 49, no. 20, pp. 32989–33003, Oct. 2023, doi: https://doi.org/10.1016/j.ceramint.2023.07.275.

[6] M. Al-Darraji, S. Jasim, O. Salah Aldeen, A. Ghasemian, and M. Rasheed, “The Effect of LL37 Antimicrobial Peptide on FOXE1 and lncRNA PTCSC 2 Genes Expression in Colorectal Cancer (CRC) and Normal Cells,” Asian Pacific Journal of Cancer Prevention, vol. 23, no. 10, pp. 3437–3442, Oct. 2022, doi: https://doi.org/10.31557/apjcp.2022.23.10.3437.

[7] Manel Sellam, M. Rasheed, S. Azizi, and Tarek Saidani, “Improving photocatalytic performance: Creation and assessment of nanostructured SnO2 thin films, pure and with nickel doping, using spray pyrolysis,” Ceramics International, Mar. 2024, doi: https://doi.org/10.1016/j.ceramint.2024.03.094.

[8] Ahcen Keziz, M. Rasheed, M. Heraiz, F. Sahnoune, and A. Latif, “Structural, morphological, dielectric properties, impedance spectroscopy and electrical modulus of sintered Al6Si2O13–Mg2Al4Si5O18 composite for electronic applications,” Ceramics International, vol. 49, no. 23, pp. 37423–37434, Dec. 2023, doi: https://doi.org/10.1016/j.ceramint.2023.09.068.

***Corresponding author**
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

[9] A. Jaber, M. Ismael, T. Rashid, Mohammed Abdulhadi Sarhan, M. Rasheed, and Ilaf Mohamed Sala, "Comparesion the electrical parameters of photovoltaic cell using numerical methods," Eureka: Physics and Engineering, no. 4, pp. 29–39, Jul. 2023, doi: https://doi.org/10.21303/2461-4262.2023.002770.
[10] N. Assoudi et al., "Comparative examination of the physical parameters of the sol gel produced compounds La0.5Ag0.1Ca0.4MnO3 and La0.6Ca0.3Ag0.1MnO3," Optical and Quantum Electronics, vol. 54, no. 9, Jul. 2022, doi: https://doi.org/10.1007/s11082-022-03927-x.
[11] M. Darraji, L. Saqban, T. Mutar, M. Rasheed, and A. Hussein, "Association of Candidate Genes Polymorphisms in Iraqi Patients with Chronic Kidney Disease," Journal of Advanced Biotechnology and Experimental Therapeutics, vol. 6, no. 1, p. 687, 2022, doi: https://doi.org/10.5455/jabet.2022.d147.
[12] M. Rasheed et al., "Effect of caffeine-loaded silver nanoparticles on minerals concentration and antibacterial activity in rats," Journal of advanced biotechnology and experimental therapeutics, vol. 6, no. 2, pp. 495–495, Jan. 2023, doi: https://doi.org/10.5455/jabet.2023.d144.
[13] Djelel Kherifi, Ahcen Keziz, M. Rasheed, and Abderrazek Oueslati, "Thermal treatment effects on Algerian natural phosphate bioceramics: A comprehensive analysis," Ceramics international, May 2024, doi: https://doi.org/10.1016/j.ceramint.2024.05.317.
[14] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, M. Rasheed, and Mohammed Abdulhadi Sarhan, "A study of the Characteristics of Wastewater on the Euphrates River in Iraq," IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022005–022005, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022005.
[15] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, and M. Rasheed, "A study of Physicochemical Parameters, Heavy Metals and Algae in the Euphrates River, Iraq," IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022007–022007, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022007.
[16] A. Zubaidi, Lamyaa Mahdi Asaad, Iqbal Alshalal, and M. Rasheed, "The impact of zirconia nanoparticles on the mechanical characteristics of 7075 aluminum alloy," Journal of the mechanical behavior of materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0302.
[17] M. Rasheed, S. Shihab, O. Alabdali, A. Rashid, and T. Rashid, "Finding Roots of Nonlinear Equation for Optoelectronic Device," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012077, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012077.
[18] M. Rasheed, O. Alabdali, S. Shihab, A. Rashid, and T. Rashid, "On the Solution of Nonlinear Equation for Photovoltaic Cell Using New Iterative Algorithms," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012078, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012078.
[19] M. Rasheed, M. Nuhad Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "The numerical Calculations of Single-Diode Solar Cell Modeling Parameters," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012058, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012058.
[20] O. Alabdali, S. Shihab, M. Rasheed, and T. Rashid, "Orthogonal Boubaker-Turki polynomials algorithm for problems arising in engineering," 3RD INTERNATIONAL SCIENTIFIC CONFERENCE OF ALKAFEEL UNIVERSITY (ISCKU 2021), 2022, doi: https://doi.org/10.1063/5.0066860.
[21] D. Bouras, M. Rasheed, R. Barille, and M. N. Aldaraji, "Efficiency of adding DD3+(Li/Mg) composite to plants and their fibers during the process of filtering solutions of toxic organic dyes," Optical Materials, vol. 131, p. 112725, Sep. 2022, doi: https://doi.org/10.1016/j.optmat.2022.112725.
[22] S. Shihab, M. Rasheed, O. Alabdali, and A. A. Abdulrahman, "A Novel Predictor-Corrector Hally Technique for Determining the Parameters for Nonlinear Solar Cell Equation," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022120, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022120.
[23] M. Rasheed, M. N. Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "Solar PV Modelling and Parameter Extraction Using Iterative Algorithms," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012059, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012059.
[24] M. Rasheed, S. Shihab, O. Y. Mohammed, and A. Al-Adili, "Parameters Estimation of Photovoltaic Model Using Nonlinear Algorithms," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012058, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012058.
[25] M. A. Sarhan, S. Shihab, B. E. Kashem, and M. Rasheed, "New Exact Operational Shifted Pell Matrices and Their Application in Astrophysics," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022122, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022122.
[26] D. Bouras, M. Fellah, A. Mecif, R. Barillé, A. Obrosov, and M. Rasheed, "High photocatalytic capacity of porous ceramic-based powder doped with MgO," Journal of the Korean Ceramic Society, Oct. 2022, doi: https://doi.org/10.1007/s43207-022-00254-5.
[27] M. Rasheed, SuhaShihab, O. Alabdali, and H. H. Hassan, "Parameters Extraction of a Single-Diode Model of Photovoltaic Cell Using False Position Iterative Method," Journal of Physics: Conference Series, vol. 1879, no. 3, p. 032113, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/3/032113.
[28] M. Rasheed, O. Y. Mohammed, S. Shihab, and A. Al-Adili, "Explicit Numerical Model of Solar Cells to Determine Current and Voltage," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012043, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012043.
[29] M. Enneffatia, M. Rasheed, B. Louatia, K. Guidaraa, S. Shihab, and R. Barillé, "Investigation of structural, morphology, optical properties and electrical transport conduction of Li0.25Na0.75CdVO4 compound," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012050, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012050.

***Corresponding author**
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com

[30] W. Saidi, Nasreddine Hfaidh, M. Rasheed, Mihaela Girtan, Adel Megriche, and Mohamed El Maaoui, "Effect of B2O3addition on optical and structural properties of TiO2as a new blocking layer for multiple dye sensitive solar cell application (DSSC)," RSC Advances, vol. 6, no. 73, pp. 68819–68826, Jan. 2016, doi: https://doi.org/10.1039/c6ra15060h.
[31] E. Kadri, M. Krichen, R. Mohammed, A. Zouari, and K. Khirouni, "Electrical transport mechanisms in amorphous silicon/crystalline silicon germanium heterojunction solar cell: impact of passivation layer in conversion efficiency," Optical and Quantum Electronics, vol. 48, no. 12, Nov. 2016, doi: https://doi.org/10.1007/s11082-016-0812-7.
[32] A. Raghdi, Menad Heraiz, M. Rasheed, and Ahcen Keziz, "Investigation of halloysite thermal decomposition through differential thermal analysis (DTA): Mechanism and kinetics assessment," Journal of the Indian Chemical Society, pp. 101413–101413, Oct. 2024, doi: https://doi.org/10.1016/j.jics.2024.101413.
[33] Ahcen Keziz, Meand Heraiz, M. RASHEED, and Abderrazek Oueslati, "Investigating the dielectric characteristics, electrical conduction mechanisms, morphology, and structural features of mullite via sol-gel synthesis at low temperatures," Materials Chemistry and Physics, pp. 129757–129757, Jul. 2024, doi: https://doi.org/10.1016/j.matchemphys.2024.129757.
[34] T. Rashid, Musa Mohd Mokji, and M. Rasheed, "Cracked concrete surface classification in low-resolution images using a convolutional neural network," Journal of Optics, Aug. 2024, doi: https://doi.org/10.1007/s12596-024-02080-w.
[35] M. Rasheed, M. N. Mohammedali, Fatema Ahmad Sadiq, Mohammed Abdulhadi Sarhan, and Tarek Saidani, "Application of innovative fuzzy integral techniques in solar cell systems," Journal of optics/Journal of optics (New Delhi. Print), Jun. 2024, doi: https://doi.org/10.1007/s12596-024-01928-5.
[36] Selma, M. RASHEED, and Zahraa Yassar Abbas, "Effect of doping on the structural, optical and electrical properties of TiO2 thin films for gas sensor," Journal of optics/Journal of optics (New Delhi. Print), May 2024, doi: https://doi.org/10.1007/s12596-024-01913-y.
[37] Farouk BOUDOU, Abdelmadjid GUENDOUZI, A. BELKREDAR, and M. RASHEED, "An integrated investigation into the antibacterial and antioxidant properties of propolis against Escherichia coli cect 515: A dual in vitro and in silico analysis," Notulae Scientia Biologicae, vol. 16, no. 2, pp. 13837–13837, May 2024, doi: https://doi.org/10.55779/nsb16211837.
[38] A. Shukur, Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "Decomposing Method for Space-Time Fractional Order PDEs," Al-Salam journal for engineering and technology, vol. 3, no. 2, pp. 1–11, May 2024, doi: https://doi.org/10.55145/ajest.2024.03.02.01.
[39] Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "The conjugate gradient approach to solve two dimensions linear elliptic boundary value equations as a prototype of the reaction diffusion system," Al-Salam journal for engineering and technology, vol. 3, no. 1, pp. 157–168, Jan. 2024, doi: https://doi.org/10.55145/ajest.2024.03.01.014.
[40] S. M. H. AL-Jawad, M. Rasheed, I. M. Ibrahim, A. S. Sabber, and A. K. Elttayf, "Impact of Copper Doping on Nanocrystalline SnO2 Thin Films Synthesized by Sol-Gel Coating and Chemical Bath Deposition for Gas Sensor Applications," Journal of nano research, vol. 84, pp. 25–40, Sep. 2024, doi: https://doi.org/10.4028/p-4frfak.

***Corresponding author**
Ruqaya Shaker Mahmood,
Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq
e-mail: ruqayashaker74@gmail.com