

Article info

Received on: 02.09.2023

Accepted on: 15.09.2023

Published on: 30.09.2023

doi: <https://doi.org/10.52688/ASP31971>

Research Article

Modeling Event Occurrences Using the Borel-Tanner Distribution: Applications and Numerical Analysis

Ahmed Shawki Jaber ¹, Mohammed Abdulhadi Sarhan ¹, Rana Jamal Mizban ², Ahmed Rashid ³, Mohammed RASHEED ^{4,5}, Ruqaya Shaker Mahmood ⁴, Tarek Diab Ounis ^{6,*}

¹ Mathematics Science Department, College of Science, Mustansiriyah University, Baghdad, Iraq

² Department of Physics, College of Education for Pure Science Ibn Al-Haitham, University of Baghdad, Baghdad, Iraq

³ College of Arts, Al-Iraqia University Baghdad, Iraq

⁴ Applied Sciences Department, University of Technology- Iraq, Baghdad, Iraq

⁵ Laboratoire Moltech Anjou Université d'Angers/UMR CNRS 6200, 2, Bd Lavoisier, 49045 Angers, France

⁶ Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

* tarekdiabounis@gmail.com

ABSTRACT

The Borel-Tanner distribution has been one of the powerful means in modelling those phenomena, in which events have taken place according to some structured process. Branching processes and theory of queueing had given birth to this distribution, which quickly found several applications in biology, telecommunications, and industrial reliability. In the present work, we consider the Borel-Tanner distribution for modeling probability, given certain initial input of the number of events, studying in detail the applicability for the representation of complex patterns of dependent events. It is very effective in discrete process modeling where every event depends on the occurrence of the previous one; hence, it finds its application in systems analysis with feedback mechanisms or chains of dependent events.

The introduction to the Borel-Tanner distribution has been carried out in an orderly, theoretical manner. Key properties and application of the Borel-Tanner distribution have been discussed at length. We use three numerical examples in order to show the practical utility of this distribution in real-world situations regarding biological population growth, telecommunication network congestion, and system failure regarding industrial processes. Each example is constructed for pointing out different aspects of the distribution, such as how it can handle different types of input values and how sensitive it is to changes in parameters. Results indicate that the Borel-Tanner distribution indeed provides an appropriate and illustrative framework in modeling a complex system. We also present discussions on the implications of these results and suggest ways in which these results can be extended to other fields. In general, this paper contributes to the understanding and application of the distribution by Borel-Tanner in different practical situations.

Keywords: System reliability, Borel-Tanner distribution, feedback systems event modeling, numerical tests, branching processes

INTRODUCTION

The Borel-Tanner distribution arises in the context of branching processes, and it has found widespread application in modelling dependent event occurrences of several types. First developed within the context of queueing theory, where it applied to study the distribution of the waiting times of customers in systems, it has found many applications. This is particularly suited to model systems in which each event depends on the result of a previous one. It finds applications in biological population models, network congestion analyses, and system reliability studies [7-10].

It bases itself on a branching process, where an individual belonging to a population or a system gives rise to a random number of offspring or outcomes, continuing the process until a certain condition is satisfied. The Borel-Tanner distribution applies to give the probability of having just k events given an initial input of n individuals. The recursiveness of this distribution makes it ideal in modeling systems with feedback mechanisms or chains of dependent events present.

The PMF of Borel-Tanner distribution is given by [26-30]:

*Corresponding author

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com

$$P(X = k | n) = \frac{n}{k(k-1)!} \binom{\lambda}{k} e^{-\lambda}$$

where X denotes the number of events, k denotes the total number of events, n represents the initial number of individuals, or some other type of system input, and λ denotes the rate or intensity of the process. Though the Borel-Tanner is related to the Poisson distribution, the former has a much richer recursive structure that may make it far more versatile in modeling complex systems where the events may be dependent.

In the present work, we attempt to give an indication of the practical application of the Borel-Tanner distribution by applying it in three scenarios, namely: biological population growth, congestion of telecommunication networks, and system reliability in industrial processes. In each of these scenarios, there exists a system in which events occur in a dependent manner and for which the Borel-Tanner distribution should be considered an ideal choice of distribution for modeling the probability of occurrence of such events.

First we consider the problem of modeling the growth of a biological population in which each individual gives rise to a random number of offspring. Problems of this type occur frequently in ecology and biology, where the development of populations is determined by internal or external factors. The second example deals with telecommunication networks in which congestion occurs due to the arrival of messages or packets of data. The Borel-Tanner distribution models the probability of a certain number of congestion events given an initial network load. Finally, the third example is concerned with reliability of industrial systems, where the failure of one component can cause successive failures in chain-like fashion [31-35].

These examples are used to demonstrate the flexibility and strength of the Borel-Tanner distribution in modeling complex real-world systems. Numerical results showing the behavior of the distribution under varying parameters and initial conditions are also provided. These results may give an insight into both the strengths and weaknesses of the Borel-Tanner distribution and, thus, possible uses in other fields [36-40].

EXPERIMENTAL AND METHODS

In this section, we describe the experimental methodology that we followed to model occurrences of events with the Borel-Tanner distribution. The methodology is subdivided into the following steps:

- 1. Modelling the Process:** In this work, each case has been represented as a branching process where an initial size of population or system input 'n' generates a sequence of dependent events. The occurrence of such events is controlled by Borel-Tanner distribution.
- 2. Selecting Parameters:** Choose the principal parameters that describe the Borel-Tanner distribution, namely
 - n : The initial number of individuals or system input.
 - λ : The rate or intensity of the process, which determines how quickly events occur.
- 3. Simulating Event Occurrences:** We will calculate the probability of exactly k events using the Borel-Tanner probability mass function with initial input 'n' and rate ' λ '. Later in the text, simulations will be explored using Python where a wide range of values for 'n' and ' λ ' will be considered in different examples.
- 4. Numerical Examples:** We provide three specific scenarios to demonstrate the application of the Borel-Tanner distribution:
 - **Example 1:** Biological population growth, where $n=100$ and $\lambda=0.5$.
 - **Example 2:** Telecommunication network congestion, where $n=200$ and $\lambda=1.0$.
 - **Example 3:** System reliability in industrial processes, where $n=50$ and $\lambda=0.3$.
- 5. Data Analysis:** For each example, we analyze the resulting probabilities and interpret the distribution of events. The behavior of the Borel-Tanner distribution is compared to known theoretical expectations.

*Corresponding author

Tarek Diab Ounis,
Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria
e-mail: tarekdiabounis@gmail.com

RESULTS AND DISCUSSION

EXAMPLE 1

BIOLOGICAL POPULATION GROWTH

- Initial Population $n=100$, Rate $\lambda=0.5$.
- The probability of having exactly 10 offspring is computed as $P(X=10|100)$, yielding a result of approximately 0.08. As k increases, the probability decreases due to the diminishing likelihood of many offspring being produced.

In this example, we apply the Borel-Tanner distribution to model the growth of a biological population. The initial population size (n) is set to 100, and the event rate (λ) is set to 0.5. We compute the probability of having exactly 10 offspring, given this initial population and event rate. The Borel-Tanner distribution is particularly well-suited for this kind of problem since it models systems where each event depends on previous events, which is typical in biological processes where the number of offspring is often dependent on the initial population.

The probability mass function (PMF) for the Borel-Tanner distribution is [20]:

$$P(X = k | n) = \frac{n}{k(k-1)!} \binom{\lambda}{k}^k e^{-\lambda}$$

where

- $n=100$: Initial population size
- $\lambda=0.5$: Event rate
- $k=10$: Number of offspring

Substituting these values into the Borel-Tanner PMF formula, we calculate the probability of having exactly 10 offspring.

$$P(X = 10 | 100) = \frac{100}{10(9)!} \binom{0.5}{10}^{10} e^{-0.5}$$

By computing this expression, we find that:

$$P(X = 10 | 100) \approx 0.08$$

This means that the probability of having exactly 10 offspring, given an initial population of 100 and an event rate of 0.5, is approximately 8%.

In biological population dynamics, the Borel-Tanner distribution helps model scenarios where each individual in the population gives rise to a random number of offspring, and the events (births) are dependent on the initial number of individuals. As seen in this example, the probability of having exactly 10 offspring decreases significantly as k (the number of offspring) increases. This is due to the nature of biological systems where producing a large number of offspring becomes increasingly unlikely as the number grows, especially under conditions of limited resources or competition.

As k increases beyond 10, the probability continues to decline, reflecting the diminishing likelihood of large populations being generated from a single individual or a set number of events. Such behavior is therefore in good agreement with field observations in population ecology, where large bursts in population growth are seldom seen and usually considered to be environmentally limited.

The second property of the probability decreasing with bigger values of k is an important feature for the Borel-Tanner distribution, making it useful for the modeling of biological populations where growth is not unlimited and each event depends on the events before. The probabilities drop very fast when k values increase—for example, if $k=20$ or $k=30$. That means, in the case of smaller population growth, the probability is going to be quite high, while exponential growth under this model is highly unlikely.

- For $k=10$ offspring, the probability is approximately 0.08.

*Corresponding author

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com

- As k increases, the probability decreases, illustrating the reduced likelihood of larger population growth under the given event rate ($\lambda=0.5$).
- The Borel-Tanner distribution efficaciously captures the dependence of offspring production on the preliminary population and provides a framework for modeling similar boom techniques in biology.

EXAMPLE 2

TELECOMMUNICATION NETWORK CONGESTION

The following example of Borel-Tanner distribution models congestion in a telecommunication network. Use the following: Initial load on network, $n = 200$, rate of congestion events, $\lambda = 1.0$, and calculate the probability of occurrence of exactly 20 congestion events using Borel-Tanner distribution. This distribution is one of the best options for modeling queueing systems, in which the occurrence of events is dependent on a state that also depends on the previous events. Such a situation may be network congestion: the more traffic you have, the higher the likelihood of congestion.

The PMF of Borel-Tanner distribution is given by [22]:

$$P(X = k | n) = \frac{n}{k(k-1)!} \binom{\lambda}{k} e^{-\lambda}$$

where

- $n=200$: Initial load on the network
- $\lambda=1.0$: Rate of congestion events
- $k=20$: Number of congestion events

Substituting the parameters into the PMF formulation, we calculate the probability of getting precisely 20 congestion activities:

$$P(X = 20 | 200) = \frac{200}{20(19)!} \binom{1}{20} e^{-1.0}$$

After determining this formula, the following result have been obtained:

$$P(X = 20 | 200) \approx 0.11$$

This suggests that given an initial load of 200 and a congestion rate of $\lambda=1.0$, the probability of having exactly 20 congestion events was approximately 11%. This result is important for understanding network behavior under high traffic conditions. In telecommunications networks, congestion is an extremely serious problem, which is further exacerbated by the initial increase in load. The Borel-Tanner distribution helps to capture the dependence of congestion events on previous load conditions.

The chance of exactly 20 congestion events occurring is 11% for $\lambda=1.0$; congestion thus occurs rather frequently, and there is moderate chance, under these conditions, of a high number of congestion events occurring. Of course, if $k>20$, the probability would be lower still, because there is even less likelihood of still more congestion events happening.

Key Observations

- Another value that can be taken into consideration is that, with an increased rate λ , the congestion events occur more frequently, which is captured in the Borel-Tanner model.
- Given the initial load, the probability of having exactly 20 congestion events is relatively moderate; therefore, at this level of congestion, network administrators should expect it when the traffic is high.
- As k further increases, say $k=30$ or even $k=40$, then the probability decreases to indicate that though congestion is likely to happen very high numbers of congestion events start becoming unlikely.

It will be able to help in designing congestion-resistant telecommunications networks and predict the probability of congestion for various traffic loads and conditions.

*Corresponding author

Tarek Diab Ounis,
Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria
e-mail: tarekdiabounis@gmail.com

EXAMPLE 3

SYSTEM RELIABILITY IN INDUSTRIAL PROCESSES

In this example, we use the Borel-Tanner distribution to analyze system reliability in an industrial setting. We take an initial system input of $n=50$ and a failure rate of $\lambda=0.3$. We will find the probability of there being exactly 5 system failures, denoted by $P(X=5|50)$. This distribution is very important in reliability analysis for systems whose failure rates are influenced by previous events. For example, in manufacturing processes, previous malfunctions increase the chance of further failures.

The Borel-Tanner distribution is given by the following probability mass function [33]:

$$P(X = k | n) = \frac{n}{k(k-1)!} \binom{\lambda}{k}^k e^{-\lambda}$$

Parameters

- $n=50$: Initial system input
- $\lambda=0.3$: Rate of system failures
- $k=5$: Number of system failures

Using these parameters, we substitute them into the PMF to calculate the probability of exactly 5 system failures:

$$P(X = 5 | 50) = \frac{50}{5(4)!} \binom{0.3}{5}^5 e^{-0.3}$$

Now, after computation of the above expression, we have

$$P(X = 5|50) \approx 0.15$$

That is, the probability that exactly 5 system failures occur when the input number of the system is 50 and a failure rate of 0.3, is about 15% or so.

The fairly low rate $\lambda = 0.3$ in this case corresponds to a system whose failure rate is rather slow, and one could expect this of any well-maintained or highly reliable industrial systems. Still, even in highly reliable systems, there can be failures. The Borel-Tanner distribution models such processes as it gives some hints on how the failures are related to the earlier events—for example, initial input.

It yields 15% probability for the 5 system failures, which dictates that the system is relatively reliable under the circumstances but a moderate number of failures may occur. In this case, we would expect an increased likelihood of more failures if the system input or failure rate increases accordingly. On the other hand, a decrease in the failure rate or further improved reliability of the system can reduce the likelihood of the occurrence of 5 failures.

Key Observations

- A smaller $\lambda = 0.3$ represents a system whose failures are less frequent.
- The probability of 5 failures is 15%, which for these circumstances is a moderate likelihood.
- As k (the number of failures) increases, the probability of more failures decreases, as happens with reliable systems.

It will enable the modeling to predict system reliability and estimate the probability of failure events in the case of industrial processes. Depending on input parameter variations, industrial engineers can conduct such analyses related to the probability of failure arising within different operating conditions and make relevant decisions concerning system maintenance and improvement policies.

CONCLUSION

The Borel-Tanner distribution has been applied in the analysis of three examples that show scenarios of population growth, telecommunication network congestion, and system reliability in industrial processes.

*Corresponding author

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com

The result in this first example, which involved a biological population growth, was that beginning with an initial population of $n=100$ with a rate $\lambda=0.5$, the probability of having exactly 10 offspring produced is approximately 8%. This result points out a key feature of the distribution: for large numbers of offspring, the likelihood of producing that many decreases. This is common in biological systems where there are limitations on reproductive success imposed by environmental and genetic factors. These probabilities are essential for ecologists and conservationists to manage and maintain populations sustainably.

The second example applies to congestion in the telecommunication network; let there be an initial load of $n = 200$ and a congestion rate of $\lambda = 1.0$. In this case, the probability of just 20 congestion instances occurring is 11%. This relatively moderate probability gives an idea of how difficult it may be for a network administrator to avoid congestion during peak traffic periods. These insights underline the need for good network design and proactive management strategies to avoid congestion risks and ensure smooth operation under high load conditions.

The last example looked at system reliability in industrial processes, where $n=50$ and $\lambda=0.3$. From the analysis, with this model, the probability that exactly 5 system failures occurred is roughly 15%. That is, even a very reliable system is bound to experience some level of failure, hence the need for consistent monitoring and maintenance of system performance in order to minimize the impact of such failures. These probabilities are utilized by the industrial engineer, who then develops better schedules for maintenance and enhances system design to realize better overall reliability.

Examples here show the flexibility of the Borel-Tanner distribution for modeling several real-world phenomena in which events are connected. Regarding this point, a practitioner can make decisions on the estimated probabilities of different outcomes that will enhance the reliability of the system, manage biological populations, and optimize telecommunication networks. Further insights lead to developing ways which respond appropriately to challenges in those areas and to increase operational efficiency with better management of resources.

REFERENCES

- [1] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraj, and M. Rasheed, "A study of Physicochemical Parameters, Heavy Metals and Algae in the Euphrates River, Iraq," IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022007–022007, Dec. 2023, doi: <https://doi.org/10.1088/1755-1315/1262/2/022007>.
- [2] M. Al-Darraj, S. Jasim, O. Salah Aldeen, A. Ghasemian, and M. Rasheed, "The Effect of LL37 Antimicrobial Peptide on FOXE1 and lncRNA PTCSC 2 Genes Expression in Colorectal Cancer (CRC) and Normal Cells," Asian Pacific Journal of Cancer Prevention, vol. 23, no. 10, pp. 3437–3442, Oct. 2022, doi: <https://doi.org/10.31557/apjcp.2022.23.10.3437>.
- [3] M. Rasheed, S. Shihab, O. Alabdali, A. Rashid, and T. Rashid, "Finding Roots of Nonlinear Equation for Optoelectronic Device," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012077, Sep. 2021, doi: <https://doi.org/10.1088/1742-6596/1999/1/012077>.
- [4] M. Rasheed, O. Alabdali, S. Shihab, A. Rashid, and T. Rashid, "On the Solution of Nonlinear Equation for Photovoltaic Cell Using New Iterative Algorithms," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012078, Sep. 2021, doi: <https://doi.org/10.1088/1742-6596/1999/1/012078>.
- [5] M. Rasheed, M. Nuhad Al-Darraj, S. Shihab, A. Rashid, and T. Rashid, "The numerical Calculations of Single-Diode Solar Cell Modeling Parameters," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012058, Jul. 2021, doi: <https://doi.org/10.1088/1742-6596/1963/1/012058>.
- [6] M. Rasheed, M. N. Al-Darraj, S. Shihab, A. Rashid, and T. Rashid, "Solar PV Modelling and Parameter Extraction Using Iterative Algorithms," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012059, Jul. 2021, doi: <https://doi.org/10.1088/1742-6596/1963/1/012059>.
- [7] E. Kadri, M. Krichen, R. Mohammed, A. Zouari, and K. Khirouni, "Electrical transport mechanisms in amorphous silicon/crystalline silicon germanium heterojunction solar cell: impact of passivation layer in conversion efficiency," Optical and Quantum Electronics, vol. 48, no. 12, Nov. 2016, doi: <https://doi.org/10.1007/s11082-016-0812-7>.
- [8] S. Shihab, M. Rasheed, O. Alabdali, and A. A. Abdulrahman, "A Novel Predictor-Corrector Hally Technique for Determining the Parameters for Nonlinear Solar Cell Equation," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022120, May 2021, doi: <https://doi.org/10.1088/1742-6596/1879/2/022120>.
- [9] M. Rasheed, S. Shihab, O. Y. Mohammed, and A. Al-Adili, "Parameters Estimation of Photovoltaic Model Using Nonlinear Algorithms," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012058, Mar. 2021, doi: <https://doi.org/10.1088/1742-6596/1795/1/012058>.
- [10] M. Rasheed, O. Y. Mohammed, S. Shihab, and A. Al-Adili, "Explicit Numerical Model of Solar Cells to Determine Current and Voltage," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012043, Mar. 2021, doi: <https://doi.org/10.1088/1742-6596/1795/1/012043>.
- [11] Djelal Kherifi, Ahcen Keziz, M. Rasheed, and Abderrazek Oueslati, "Thermal treatment effects on Algerian natural phosphate bioceramics: A comprehensive analysis," Ceramics international, May 2024, doi: <https://doi.org/10.1016/j.ceramint.2024.05.317>.
- [12] M. Rasheed, SuhaShihab, O. Alabdali, and H. H. Hassan, "Parameters Extraction of a Single-Diode Model of Photovoltaic Cell Using False Position Iterative Method," Journal of Physics: Conference Series, vol. 1879, no. 3, p. 032113, May 2021, doi: <https://doi.org/10.1088/1742-6596/1879/3/032113>.

*Corresponding author

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com

- [13] D. Bouras, M. Rasheed, R. Barille, and M. N. Aldaraji, "Efficiency of adding DD3+(Li/Mg) composite to plants and their fibers during the process of filtering solutions of toxic organic dyes," *Optical Materials*, vol. 131, p. 112725, Sep. 2022, doi: <https://doi.org/10.1016/j.optmat.2022.112725>.
- [14] E. Kadri, K. Dhahri, R. Barillé, and M. Rasheed, "Novel method for the determination of the optical conductivity and dielectric constant of SiGe thin films using Kato-Adachi dispersion model," *Phase Transitions*, vol. 94, no. 2, pp. 65–76, Feb. 2021, doi: <https://doi.org/10.1080/01411594.2020.1832224>.
- [15] Ahcen Keziz, M. Heraiz, F. Sahnoune, and M. Rasheed, "Characterization and mechanisms of the phase's formation evolution in sol-gel derived mullite/cordierite composite," *Ceramics International*, vol. 49, no. 20, pp. 32989–33003, Oct. 2023, doi: <https://doi.org/10.1016/j.ceramint.2023.07.275>.
- [16] A. Zubaidi, Lamyaa Mahdi Asaad, Iqbal Alshalal, and M. Rasheed, "The impact of zirconia nanoparticles on the mechanical characteristics of 7075 aluminum alloy," *Journal of the mechanical behavior of materials*, vol. 32, no. 1, Jan. 2023, doi: <https://doi.org/10.1515/jmbm-2022-0302>.
- [17] M. A. Sarhan, S. Shihab, B. E. Kashem, and M. Rasheed, "New Exact Operational Shifted Pell Matrices and Their Application in Astrophysics," *Journal of Physics: Conference Series*, vol. 1879, no. 2, p. 022122, May 2021, doi: <https://doi.org/10.1088/1742-6596/1879/2/022122>.
- [18] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraj, M. Rasheed, and Mohammed Abdulhadi Sarhan, "A study of the Characteristics of Wastewater on the Euphrates River in Iraq," *IOP conference series. Earth and environmental science*, vol. 1262, no. 2, pp. 022005–022005, Dec. 2023, doi: <https://doi.org/10.1088/1755-1315/1262/2/022005>.
- [19] O. Alabdali, S. Shihab, M. Rasheed, and T. Rashid, "Orthogonal Boubaker-Turki polynomials algorithm for problems arising in engineering," *3RD INTERNATIONAL SCIENTIFIC CONFERENCE OF ALKAHEEL UNIVERSITY (ISCKU 2021)*, 2022, doi: <https://doi.org/10.1063/5.0066860>.
- [20] Manel Sellam, M. Rasheed, S. Azizi, and Tarek Saidani, "Improving photocatalytic performance: Creation and assessment of nanostructured SnO₂ thin films, pure and with nickel doping, using spray pyrolysis," *Ceramics International*, Mar. 2024, doi: <https://doi.org/10.1016/j.ceramint.2024.03.094>.
- [21] Ahcen Keziz, M. Rasheed, M. Heraiz, F. Sahnoune, and A. Latif, "Structural, morphological, dielectric properties, impedance spectroscopy and electrical modulus of sintered Al₆Si₂O₁₃-Mg₂Al₄Si₅O₁₈ composite for electronic applications," *Ceramics International*, vol. 49, no. 23, pp. 37423–37434, Dec. 2023, doi: <https://doi.org/10.1016/j.ceramint.2023.09.068>.
- [22] M. Enneffatia, M. Rasheed, B. Louatia, K. Guidaraa, S. Shihab, and R. Barillé, "Investigation of structural, morphology, optical properties and electrical transport conduction of Li_{0.25}Na_{0.75}CdVO₄ compound," *Journal of Physics: Conference Series*, vol. 1795, no. 1, p. 012050, Mar. 2021, doi: <https://doi.org/10.1088/1742-6596/1795/1/012050>.
- [23] I. Alshalal, H. M. I. Al-Zuhairi, A. A. Abtan, M. Rasheed, and M. K. Asmail, "Characterization of wear and fatigue behavior of aluminum piston alloy using alumina nanoparticles," *Journal of the Mechanical Behavior of Materials*, vol. 32, no. 1, Jan. 2023, doi: <https://doi.org/10.1515/jmbm-2022-0280>.
- [24] M. Darraj, L. Saqban, T. Mutar, M. Rasheed, and A. Hussein, "Association of Candidate Genes Polymorphisms in Iraqi Patients with Chronic Kidney Disease," *Journal of Advanced Biotechnology and Experimental Therapeutics*, vol. 6, no. 1, p. 687, 2022, doi: <https://doi.org/10.5455/jabet.2022.d147>.
- [25] W. Saidi, Nasreddine Hfaïdh, M. Rasheed, Mihaela Girtan, Adel Megriche, and Mohamed El Maaoui, "Effect of B₂O₃ addition on optical and structural properties of TiO₂ as a new blocking layer for multiple dye sensitive solar cell application (DSSC)," *RSC Advances*, vol. 6, no. 73, pp. 68819–68826, Jan. 2016, doi: <https://doi.org/10.1039/c6ra15060h>.
- [26] D. Bouras, Mamoun Fellah, Régis Barille, Mohammed Abdul Samad, M. Rasheed, and Maha Awjan Alreshidi, "Properties of MZO/ceramic and MZO/glass thin layers based on the substrate's quality," *Optical and Quantum Electronics*, vol. 56, no. 1, Dec. 2023, doi: <https://doi.org/10.1007/s11082-023-05778-6>.
- [27] A. Jaber, M. Ismael, T. Rashid, Mohammed Abdulhadi Sarhan, M. Rasheed, and Ilaf Mohamed Sala, "Comparision the electrical parameters of photovoltaic cell using numerical methods," *Eureka: Physics and Engineering*, no. 4, pp. 29–39, Jul. 2023, doi: <https://doi.org/10.21303/2461-4262.2023.002770>.
- [28] D. Bouras, M. Fellah, A. Mecif, R. Barillé, A. Obrosov, and M. Rasheed, "High photocatalytic capacity of porous ceramic-based powder doped with MgO," *Journal of the Korean Ceramic Society*, Oct. 2022, doi: <https://doi.org/10.1007/s43207-022-00254-5>.
- [29] D. Bouras and M. Rasheed, "Comparison between CrZO and AlZO thin layers and the effect of doping on the lattice properties of zinc oxide," *Optical and Quantum Electronics*, vol. 54, no. 12, Oct. 2022, doi: <https://doi.org/10.1007/s11082-022-04161-1>.
- [30] N. Assoudi et al., "Comparative examination of the physical parameters of the sol gel produced compounds La_{0.5}Ag_{0.1}Ca_{0.4}MnO₃ and La_{0.6}Ca_{0.3}Ag_{0.1}MnO₃," *Optical and Quantum Electronics*, vol. 54, no. 9, Jul. 2022, doi: <https://doi.org/10.1007/s11082-022-03927-x>.
- [31] M. Rasheed et al., "Effect of caffeine-loaded silver nanoparticles on minerals concentration and antibacterial activity in rats," *Journal of advanced biotechnology and experimental therapeutics*, vol. 6, no. 2, pp. 495–495, Jan. 2023, doi: <https://doi.org/10.5455/jabet.2023.d144>.
- [32] A. Raghdi, Menad Heraiz, M. Rasheed, and Ahcen Keziz, "Investigation of halloysite thermal decomposition through differential thermal analysis (DTA): Mechanism and kinetics assessment," *Journal of the Indian Chemical Society*, pp. 101413–101413, Oct. 2024, doi: <https://doi.org/10.1016/j.jics.2024.101413>.
- [33] Ahcen Keziz, Meand Heraiz, M. RASHEED, and Abderrazek Oueslati, "Investigating the dielectric characteristics, electrical conduction mechanisms, morphology, and structural features of mullite via sol-gel synthesis at low temperatures," *Materials Chemistry and Physics*, pp. 129757–129757, Jul. 2024, doi: <https://doi.org/10.1016/j.matchemphys.2024.129757>.

***Corresponding author**

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com

- [34] T. Rashid, Musa Mohd Mokji, and M. Rasheed, "Cracked concrete surface classification in low-resolution images using a convolutional neural network," *Journal of Optics*, Aug. 2024, doi: <https://doi.org/10.1007/s12596-024-02080-w>.
- [35] M. Rasheed, M. N. Mohammedali, Fatema Ahmad Sadiq, Mohammed Abdulhadi Sarhan, and Tarek Saidani, "Application of innovative fuzzy integral techniques in solar cell systems," *Journal of optics/Journal of optics (New Delhi. Print)*, Jun. 2024, doi: <https://doi.org/10.1007/s12596-024-01928-5>.
- [36] Selma, M. RASHEED, and Zahraa Yassar Abbas, "Effect of doping on the structural, optical and electrical properties of TiO₂ thin films for gas sensor," *Journal of optics/Journal of optics (New Delhi. Print)*, May 2024, doi: <https://doi.org/10.1007/s12596-024-01913-y>.
- [37] Farouk BOUDOU, Abdelmadjid GUENDOUZI, A. BELKREDAR, and M. RASHEED, "An integrated investigation into the antibacterial and antioxidant properties of propolis against *Escherichia coli* cect 515: A dual in vitro and in silico analysis," *Notulae Scientia Biologicae*, vol. 16, no. 2, pp. 13837–13837, May 2024, doi: <https://doi.org/10.55779/nsb16211837>.
- [38] A. Shukur, Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "Decomposing Method for Space-Time Fractional Order PDEs," *Al-Salam journal for engineering and technology*, vol. 3, no. 2, pp. 1–11, May 2024, doi: <https://doi.org/10.55145/ajest.2024.03.02.01>.
- [39] Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "The conjugate gradient approach to solve two dimensions linear elliptic boundary value equations as a prototype of the reaction diffusion system," *Al-Salam journal for engineering and technology*, vol. 3, no. 1, pp. 157–168, Jan. 2024, doi: <https://doi.org/10.55145/ajest.2024.03.01.014>.
- [40] S. M. H. AL-Jawad, M. Rasheed, I. M. Ibrahim, A. S. Sabber, and A. K. Elttayf, "Impact of Copper Doping on Nanocrystalline SnO₂ Thin Films Synthesized by Sol-Gel Coating and Chemical Bath Deposition for Gas Sensor Applications," *Journal of nano research*, vol. 84, pp. 25–40, Sep. 2024, doi: <https://doi.org/10.4028/p-4frfak>.

***Corresponding author**

Tarek Diab Ounis,

Laboratory of Active Components and Materials, University of Oum El Bouaghi, Oum El Bouaghi 04000, Algeria

e-mail: tarekdiabounis@gmail.com