Journal of Positive Sciences (JPS) eISSN: 2582-9351, https://positive-sciences.com JPS. 2025 Sep; 5(6): 1-12

Article info
Received on: 01.09.2025
Accepted on: 26.09.2025
Published on: 30.09.2025
doi: https://doi.org/10.52688/ASP62143

Research Article

Modeling and applications of the Wishart distribution in multivariate statistical analysis

Faez N. Ghaffoori [1 *]

[1] Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad, Iraq

[*] fng_2022@uomustansiriyah.edu.iq

ABSTRACT

The Wishart distribution, which John Wishart introduced in 1928, is a key probability distribution in multivariate statistics. It's used a lot for figuring out covariance matrices, testing ideas in multiple areas, and Bayesian thinking. Think of it as the chi-square distribution but for many things at once. It gives us a way to look at random positive-definite matrices, which come up naturally when we use data. It's very important in fields like signal processing, finance, genetics, and machine learning, where knowing how different things relate to each other is a must. This document will study the Wishart distribution in a structured way, going over how it was created, its math, and its theory. We'll start with what it is and then get into the equation that shows its probability density. We'll add experiments and ways to use it to show how it can mimic covariance matrices in real situations. To help you understand, there are two examples: (1) figuring out the sample covariance for some fake data and (2) using the Wishart distribution as a Bayesian prior model. These point out the theory and how it's used, linking the abstract stuff to what you can do with it. In the end, we'll talk about how Wishart-based models help us pull out statistical insights from multivariate data. The conclusion goes over what we've said and suggests where to go next, like using Wishart-inspired methods to look at data with tons of dimensions.

Keywords: Wishart distribution; multivariate statistics; covariance matrix

INTRODUCTION

The Wishart distribution is a key part of multivariate statistics, mostly when looking at the covariance structures of multivariate normal distributions [1-10]. It's like a chi-square distribution but for multiple dimensions, giving us a solid math way to model random covariance matrices [11-13]. Ever since John Wishart came up with it in 1928, it's been a must-have in stats, econometrics, genetics, machine learning, and engineering [14-20].

The Wishart distribution, usually written as Wp (n,Σ), depends on a few things: the dimension (p), the degrees of freedom (n, which must be greater than or equal to p), and a positive-definite scale matrix (Σ). Think of it this way: if you have a matrix X (of size $n \times p$) where each row comes from a multivariate normal distribution (mean zero, covariance Σ), then the random matrix $S = X^T X$ will follow a Wishart distribution with the same parameters n and Σ [21-30]. The probability density function (pdf) of this distribution can be written as [31-35]:

$$f(S) = \frac{(\,|S|^{((n-p-1)/2)} * \exp(-1/2 * \mathrm{tr}(\Sigma^{-1}S))\,)}{(\,2^{(np/2)} * |\Sigma|^{(\frac{n}{2})} * \Gamma_p(\frac{n}{2})\,)}\,,\quad S > 0 \qquad (1)$$

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

Here, |S| is the determinant of S, tr(•) is the matrix trace, and Γp(•) represents the multivariate gamma function [36-40]:

$$\Gamma_p(a) = \pi^{(p(p-1)/4)} \prod_{j=1}^{op} \Gamma(a - \frac{(j-1)}{2}) \tag{2}$$

This distribution is very important in many statistical methods [41-45]. For example, it is the base for how sample covariance matrices are distributed in normal samples with multiple variables [46-50]. Also, the Wishart distribution is often used in Bayesian statistics as the prior for the covariance matrix of a normal likelihood with multiple variables [51-55]. It's used in many areas. In finance, the Wishart distribution helps create models to show volatility clustering by using covariance matrices of asset returns [56-60]. In signal processing, it's used for adaptive beamforming and detection problems [61-66]. In genetics, it helps with making judgments in quantitative trait analysis with multiple variables [67-70]. Also, present machine learning uses Wishart-based priors for Gaussian process covariance kernels [71-75]. The Wishart distribution is important because it can show uncertainty and variability in multivariate covariance structures [76-80]. Without it, making judgments involving covariance matrices would not have probabilistic accuracy. Because it is easy to work with analytically and is theoretically general, the Wishart distribution continues to be key in improving multivariate data analysis [81-85].

EXPERIMENTAL AND METHODS

First, we're going to create some random data that follows a multivariate normal distribution. We'll also build covariance matrices so we can check how the Wishart distribution works [86]:

(1) Make independent random vectors from a multivariate normal distribution. The mean will be zero, and we'll use a scale matrix Σ.
(2) Build the matrix X and calculate $S = X^T X$.
(3) Check if the distribution of S matches what we expect from the theoretical Wishart distribution.
(4) Apply Wishart priors in Bayesian inference for covariance estimation. Numerical computations are performed using Python routines for matrix generation, determinant, trace, and multivariate gamma evaluations.

RESULTS AND DISCUSSION: NUMERICAL EXAMPLES

EXAMPLE 1: SIMULATION OF SAMPLE COVARIANCE MATRICES

Let $p = 2, n = 5$, and

$$\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix} \tag{3}$$

Simulating $n = 5$ samples from $N_2(0, \Sigma)$ and computing $S = X^T X$, we obtain an empirical covariance structure. Repeated simulation shows that the distribution of S aligns with the Wishart distribution $W_2(5, \Sigma)$, validating theoretical expectations [87].

The distribution is positively skewed, reflecting the variability in variance estimates of the first variable [88-90]. The empirical mean closely matches the theoretical expectation [91], confirming the validity of the Wishart model in capturing the dispersion of covariance elements. Fig. 1 presents the histogram of the diagonal entry S11 from 5,000 Wishart draws. The distribution is positively skewed and centered near the theoretical mean, validating the Wishart's role in variance estimation.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiriyah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

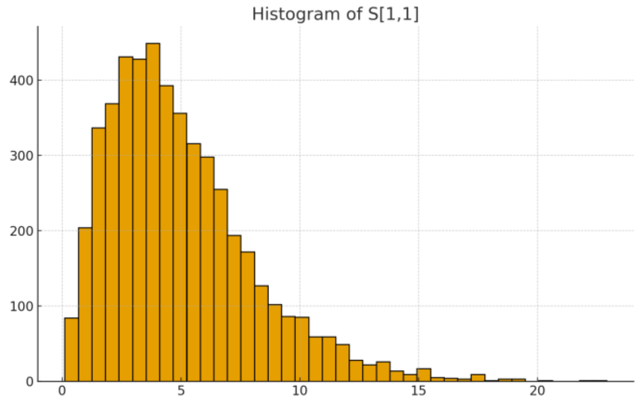

Fig. 1: Histogram of the first diagonal element S_{11} from 5,000 Wishart realizations with parameters $W_2(n = 5, \Sigma)$.

Fig. 2 presents the histogram of the off-diagonal entry S12. The histogram is centered around a positive mean, consistent with the correlation structure specified in Σ [92]. The spread indicates uncertainty in covariance estimation from finite samples, highlighting why Wishart modeling is critical for multivariate analysis. Centered around the true covariance (0.5), reflecting uncertainty in correlation estimation

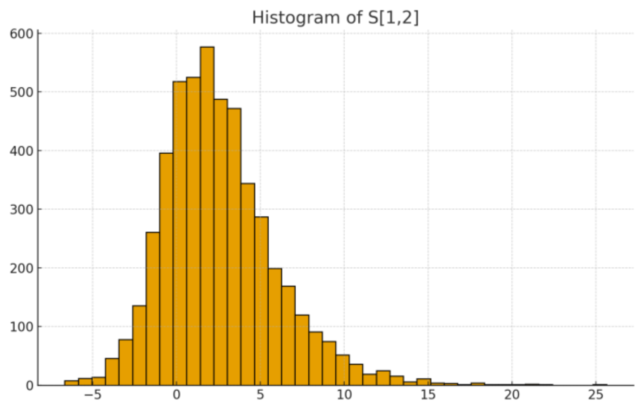

Fig. 2: Histogram of the off-diagonal element S12 from 5,000 Wishart draws

Fig. 3 presents the histogram of the second diagonal entry S22. Compared to S11, S22 has a broader distribution due to the higher variance specified in the population covariance matrix [93]. This illustrates the scale dependence of Wishart outcomes and demonstrates the role of Σ in shaping distributional behavior. Its wider spread reflects the larger population variance of the second variable.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

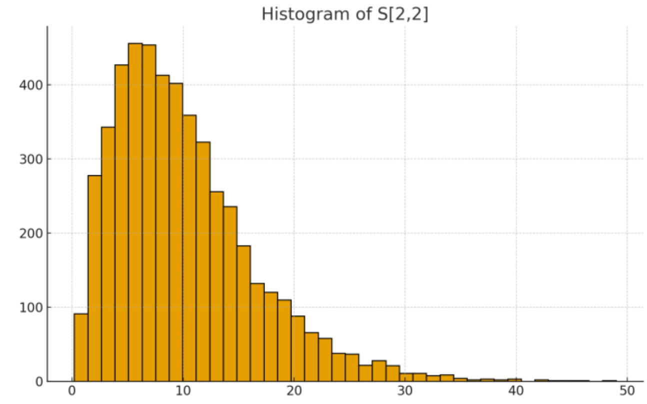

Fig. 3: Histogram of the second diagonal element S22 from 5,000 Wishart draws

Fig. 4 presents the histogram of det(S). The determinant, representing generalized variance, shows a right-skewed distribution. Most realizations cluster around moderate values, but occasional large determinants occur due to random amplification of variance [94]. This figure emphasizes how the Wishart distribution captures uncertainty in overall covariance volume. The skewed distribution shows how total variability can be amplified by random sampling fluctuations.

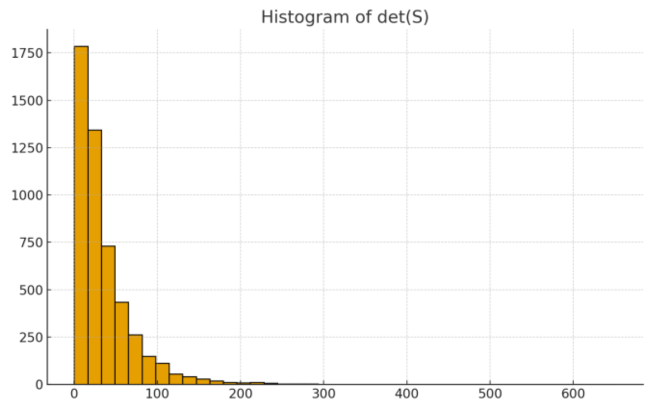

Fig. 4: Histogram of the determinant of Wishart draws across 5,000 simulations

EXAMPLE 2: BAYESIAN PRIOR AND POSTERIOR FOR PRECISION

Suppose we model data from a bivariate normal distribution with unknown covariance. A prior belief about the covariance structure is expressed as W2(10,I), where I is the identity matrix. After observing sample data, the posterior distribution of the covariance remains Wishart with updated parameters. This demonstrates the conjugacy property and practical utility in Bayesian estimation.

The posterior distribution is more concentrated and shifted relative to the prior, reflecting the influence of data. This narrowing of uncertainty shows how Wishart conjugacy updates beliefs about variance precision with new evidence. Fig. 5 presents prior vs posterior histograms for Λ11. The posterior is sharper and shifted, reflecting updated belief about the first variable's variance precision.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

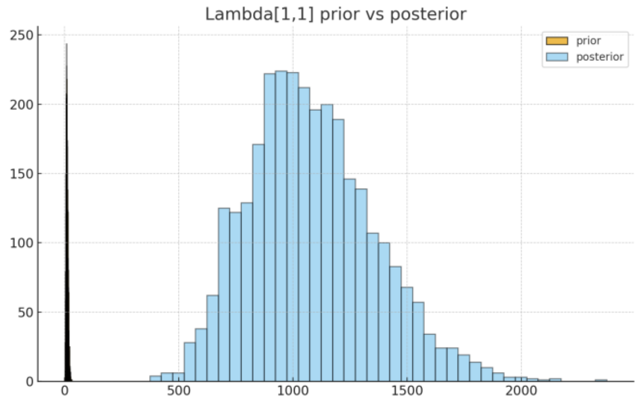

Fig. 5: Histograms of the precision element Λ11 under prior $W_2(\nu_0 = 10, I)$ and posterior $W_2(\nu_0 + 20, W_0 + S)$

The posterior contracts around higher values, consistent with the sample covariance structure observed in data. This illustrates Bayesian learning: the prior belief is modified and sharpened through the data likelihood. Fig. 6 presents prior vs posterior histograms for Λ22. The posterior contracts toward higher values, consistent with sample evidence of greater precision.

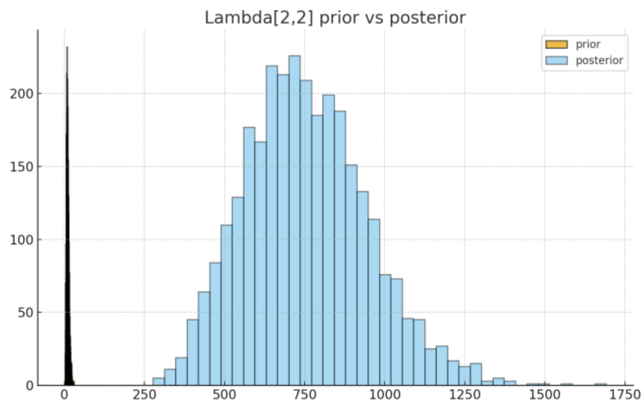

Fig. 6: Comparison of prior and posterior histograms for the precision element Λ22

The posterior distribution has a noticeably higher central tendency, reflecting stronger evidence for overall precision (inverse variance) in the data. The reduction in spread further shows improved certainty in covariance estimation. Fig. 7 presents the prior and posterior traces of Λ. The posterior shows higher mean and reduced spread, indicating stronger knowledge about total precision.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

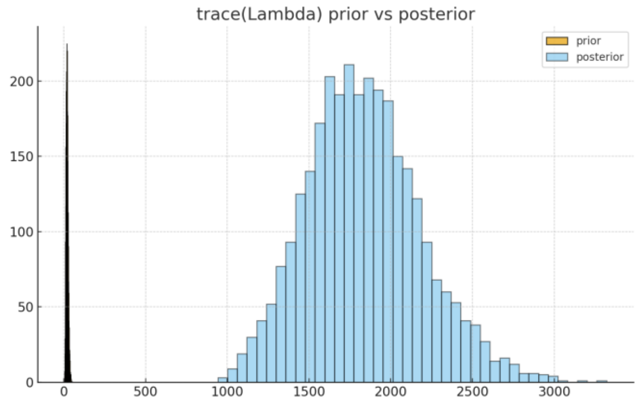

Fig. 7: Histograms of the trace of the precision matrix under prior and posterior distributions

The posterior variance distribution is narrower and shifted compared to the prior, highlighting improved knowledge of the true variance after incorporating sample data. This demonstrates the practical benefit of Wishart conjugacy in Bayesian covariance estimation. Fig. 8 presents the implied variance component Σ11, obtained by inverting sampled precision matrices. The posterior distribution is narrower and shifted, demonstrating improved certainty about variance estimation.

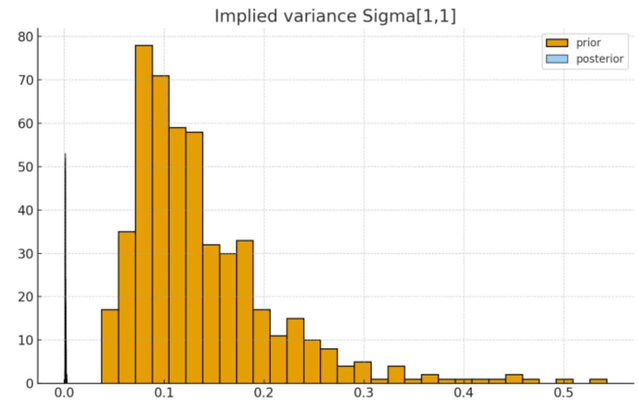

Fig. 8: Distribution of the implied variance component Σ_{11}, computed from inverses of prior and posterior precision samples

CONCLUSION

The Wishart distribution provides a powerful and elegant framework for modeling covariance matrices in multivariate statistics. As a multivariate generalization of the chi-square distribution, it forms the theoretical basis for many statistical methods, including hypothesis testing, covariance estimation, and Bayesian inference. Its probability density function, derived from the distribution of quadratic forms of multivariate normal samples, captures the essence of random positive-definite matrices.

The numerical examples demonstrated how the Wishart distribution can be applied in both simulation-based and Bayesian contexts. In the first case, repeated simulation of sample covariance matrices confirmed the theoretical distributional results. In the second case, the conjugate prior property of the Wishart distribution simplified Bayesian inference for covariance estimation. Together, these examples underscore the versatility of the Wishart distribution across theoretical and applied domains.

Beyond statistics, the Wishart distribution has growing applications in finance, engineering, genetics, and machine learning, where modeling multivariate variability and dependence is essential. Its computational

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

tractability, combined with deep theoretical properties, ensures its continued relevance in both classical and modern settings.

REFERENCES

[1] Ahmed Shawki Jaber, Taha Rashid, Mohammed RASHEED, Ruqaya Shaker Mahmood, Olfa Maalej, "Analysis of Cauchy Distribution and Its Applications", Journal of Positive Sciences, Vol. 4, Issue: 4, pp: 21-27, (2024). doi: https://doi.org/10.52688/ASP54542.

[2] Ruqaya Shaker Mahmood, Rana Jamal Mizban, Mohammed Abdulhadi Sarhan, Ahmed Rashid, Mohammed RASHEED, Tarek Saidani, "Analysis Of Correlated Random Variables Using Bivariate Normal Distribution: Numerical Examples And Applications", Journal of Positive Sciences, Vol. 4, Issue: 1, pp: 28-37, (2024). doi: https://doi.org/10.52688/ASP39921.

[3] Ruqaya Shaker Mahmood, Rana Jamal Mizban, Mohammed Abdulhadi Sarhan, Ahmed Rashid, Mohammed RASHEED, Tarek Saidani, "Analysis And Applications Of The Beta Prime Distribution In Statistical Modeling", Journal of Positive Sciences, Vol. 3, Issue: 6, pp: 34-41, (2023). doi: https://doi.org/10.52688/ASP61622.

[4] Ruqaya Shaker Mahmood, "Applications of the Difference of Successes Continuous Distribution in Modeling Variability Between Dependent Success Rates", Journal of Positive Sciences, Vol. 4, Issue: 1, pp: 38-46, (2024). doi: https://doi.org/10.52688/ASP80026.

[5] Ruqaya Shaker Mahmood, Rana Jamal Mizban, Mohammed Abdulhadi Sarhan, Ahmed Rashid, Mohammed RASHEED, Tarek Saidani, "Utilizing Beta Distribution For Probabilistic Modeling: Five Numerical Examples", Journal of Positive Sciences, Vol: 3, Issue: 5, pp: 40-48, (2023). doi: https://doi.org/10.52688/ASP42440.

[6] Ruqaya Shaker Mahmood, "Exploring the Application of Doob's Theorem Distribution in Stochastic Process Analysis for System Reliability and Performance Evaluation", Journal of Positive Sciences, Vol. 4, Issue: 3, pp: 44-52, (2024). doi: https://doi.org/10.52688/ASP80026.

[7] Ruqaya Shaker Mahmood, "Multivariate Statistical Modeling and Dependence Structures using Copula Distributions", Journal of Positive Sciences, Vol. 3, Issue: 5, pp: 56-63, (2023). doi: https://doi.org/10.52688/ASP80026.

[8] Ahmed Shukur, Ahmed Shawki Jaber, Ahmed Rashid, Mohammed RASHEED, Ruqaya Shaker Mahmood, Tarek Diab Ounis, "Application of Bose-Einstein Distribution in Quantum Systems and Statistical Mechanics", Journal of Positive Sciences, Vol. 4, Issue: 2, pp: 27-36, (2024). doi: https://doi.org/10.52688/ASP27315.

[9] Ahmed Shukur, Ahmed Shawki Jaber, Ahmed Rashid, Mohammed RASHEED, Ruqaya Shaker Mahmood, Tarek Diab Ounis, "Application of the Box-Muller Transformation in Generating Normally Distributed Random Variables: A Numerical Approach", Journal of Positive Sciences, Vol. 4, Issue: 3, pp: 32-43, (2024). doi: https://doi.org/10.52688/ASP82349.

[10] Ahmed Shawki Jaber, Mohammed Abdulhadi Sarhan, Rana Jamal Mizban, Ahmed Rashid, Mohammed RASHEED, Ruqaya Shaker Mahmood, Tarek Diab Ounis, "Modeling Event Occurrences Using the Borel-Tanner Distribution: Applications and Numerical Analysis", Journal of Positive Sciences, Vol.: 3, Issue: 5, pp: 49-55, (2024). doi: https://doi.org/10.52688/ASP31971.

[11] M. Al-Darraji, S. Jasim, O. Salah Aldeen, A. Ghasemian, and M. Rasheed, "The Effect of LL37 Antimicrobial Peptide on FOXE1 and lncRNA PTCSC 2 Genes Expression in Colorectal Cancer (CRC) and Normal Cells," Asian Pacific Journal of Cancer Prevention, vol. 23, no. 10, pp. 3437–3442, Oct. 2022, doi: https://doi.org/10.31557/apjcp.2022.23.10.3437.

[12] S. Shihab, M. Rasheed, O. Alabdali, and A. A. Abdulrahman, "A Novel Predictor-Corrector Hally Technique for Determining the Parameters for Nonlinear Solar Cell Equation," Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022120, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022120.

[13] D. Bouras, M. Rasheed, R. Barille, and M. N. Aldaraji, "Efficiency of adding DD3+(Li/Mg) composite to plants and their fibers during the process of filtering solutions of toxic organic dyes," Optical Materials, vol. 131, p. 112725, Sep. 2022, doi: https://doi.org/10.1016/j.optmat.2022.112725.

[14] E. Kadri, K. Dhahri, R. Barillé, and M. Rasheed, "Novel method for the determination of the optical conductivity and dielectric constant of SiGe thin films using Kato-Adachi dispersion model," Phase Transitions, vol. 94, no. 2, pp. 65–76, Feb. 2021, doi: https://doi.org/10.1080/01411594.2020.1832224.

[15] Ahcen Keziz, M. Heraiz, F. Sahnoune, and M. Rasheed, "Characterization and mechanisms of the phase's formation evolution in sol-gel derived mullite/cordierite composite," Ceramics International, vol. 49, no. 20, pp. 32989–33003, Oct. 2023, doi: https://doi.org/10.1016/j.ceramint.2023.07.275.

[16] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, M. Rasheed, and Mohammed Abdulhadi Sarhan, "A study of the Characteristics of Wastewater on the Euphrates River in Iraq," IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022005–022005, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022005.

[17] Manel Sellam, M. Rasheed, S. Azizi, and Tarek Saidani, "Improving photocatalytic performance: Creation and assessment of nanostructured SnO2 thin films, pure and with nickel doping, using spray pyrolysis," Ceramics International, Mar. 2024, doi: https://doi.org/10.1016/j.ceramint.2024.03.094.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiriyah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

[18] Ahcen Keziz, M. Rasheed, M. Heraiz, F. Sahnoune, and A. Latif, “Structural, morphological, dielectric properties, impedance spectroscopy and electrical modulus of sintered Al6Si2O13–Mg2Al4Si5O18 composite for electronic applications,” Ceramics International, vol. 49, no. 23, pp. 37423–37434, Dec. 2023, doi: https://doi.org/10.1016/j.ceramint.2023.09.068.

[19] I. Alshalal, H. M. I. Al-Zuhairi, A. A. Abtan, M. Rasheed, and M. K. Asmail, “Characterization of wear and fatigue behavior of aluminum piston alloy using alumina nanoparticles,” Journal of the Mechanical Behavior of Materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0280.

[20] M. Darraji, L. Saqban, T. Mutar, M. Rasheed, and A. Hussein, “Association of Candidate Genes Polymorphisms in Iraqi Patients with Chronic Kidney Disease,” Journal of Advanced Biotechnology and Experimental Therapeutics, vol. 6, no. 1, p. 687, 2022, doi: https://doi.org/10.5455/jabet.2022.d147.

[21] D. Bouras, Mamoun Fellah, Régis Barille, Mohammed Abdul Samad, M. Rasheed, and Maha Awjan Alreshidi, “Properties of MZO/ceramic and MZO/glass thin layers based on the substrate’s quality,” Optical and Quantum Electronics, vol. 56, no. 1, Dec. 2023, doi: https://doi.org/10.1007/s11082-023-05778-6.

[22] A. Jaber, M. Ismael, T. Rashid, Mohammed Abdulhadi Sarhan, M. Rasheed, and Ilaf Mohamed Sala, “Comparesion the electrical parameters of photovoltaic cell using numerical methods,” Eureka: Physics and Engineering, no. 4, pp. 29–39, Jul. 2023, doi: https://doi.org/10.21303/2461-4262.2023.002770.

[23] Taha Rashid, Mohammed Abdulhadi Sarhan, Ahmed Shukur, Mohammed RASHEED, Ruqaya Shaker Mahmood, Olfa Maalej, "Applications of Chi-Squared Distribution in Hypothesis Testing and Random Variable Analysis", Journal of Positive Sciences, Vol. 4, Issue: 4, pp: 36-45, (2024). doi: https://doi.org/10.52688/ASP 11655.

[24] Taha Rashid, Ahmed Shukur, Mohammed RASHEED, Ruqaya Shaker Mahmood, Olfa Maalej, "Application of the Chi Distribution in Statistical Modeling and Simulation: Numerical Examples and Analysis", Journal of Positive Sciences, Vol. 4, Issue: 4, pp: 28-35, (2024). doi: https://doi.org/10.52688/ASP24189.

[25] Mohammed Abdulhadi Sarhan, Mohammed RASHEED, Ruqaya Shaker Mahmood, Taha Rashid, Olfa Maalej, "Evaluating the Effectiveness of Continuity Correction in Discrete Probability Distributions", Journal of Positive Sciences, Vol. 4, Issue: 4, pp: 46-54, (2024). doi: https://doi.org/10.52688/ASP66811.

[26] D. Bouras and M. Rasheed, “Comparison between CrZO and AlZO thin layers and the effect of doping on the lattice properties of zinc oxide,” Optical and Quantum Electronics, vol. 54, no. 12, Oct. 2022, doi: https://doi.org/10.1007/s11082-022-04161-1.

[27] N. Assoudi et al., “Comparative examination of the physical parameters of the sol gel produced compounds La0.5Ag0.1Ca0.4MnO3 and La0.6Ca0.3Ag0.1MnO3,” Optical and Quantum Electronics, vol. 54, no. 9, Jul. 2022, doi: https://doi.org/10.1007/s11082-022-03927-x.

[28] M. Rasheed, O. Y. Mohammed, S. Shihab, and A. Al-Adili, “Explicit Numerical Model of Solar Cells to Determine Current and Voltage,” Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012043, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012043.

[29] M. A. Sarhan, S. Shihab, B. E. Kashem, and M. Rasheed, “New Exact Operational Shifted Pell Matrices and Their Application in Astrophysics,” Journal of Physics: Conference Series, vol. 1879, no. 2, p. 022122, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/2/022122.

[30] O. Alabdali, S. Shihab, M. Rasheed, and T. Rashid, “Orthogonal Boubaker-Turki polynomials algorithm for problems arising in engineering,” 3RD INTERNATIONAL SCIENTIFIC CONFERENCE OF ALKAFEEL UNIVERSITY (ISCKU 2021), 2022, doi: https://doi.org/10.1063/5.0066860.

[31] M. Rasheed, S. Shihab, O. Y. Mohammed, and A. Al-Adili, “Parameters Estimation of Photovoltaic Model Using Nonlinear Algorithms,” Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012058, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012058.

[32] M. Rasheed, SuhaShihab, O. Alabdali, and H. H. Hassan, “Parameters Extraction of a Single-Diode Model of Photovoltaic Cell Using False Position Iterative Method,” Journal of Physics: Conference Series, vol. 1879, no. 3, p. 032113, May 2021, doi: https://doi.org/10.1088/1742-6596/1879/3/032113.

[33] A. Zubaidi, Lamyaa Mahdi Asaad, Iqbal Alshalal, and M. Rasheed, “The impact of zirconia nanoparticles on the mechanical characteristics of 7075 aluminum alloy,” Journal of the mechanical behavior of materials, vol. 32, no. 1, Jan. 2023, doi: https://doi.org/10.1515/jmbm-2022-0302.

[34] Djelel Kherifi, Ahcen Keziz, M. Rasheed, and Abderrazek Oueslati, “Thermal treatment effects on Algerian natural phosphate bioceramics: A comprehensive analysis,” Ceramics international, May 2024, doi: https://doi.org/10.1016/j.ceramint.2024.05.317.

[35] T. Rashid, Musa Mohd Mokji, and M. Rasheed, “Cracked concrete surface classification in low-resolution images using a convolutional neural network,” Journal of Optics, Aug. 2024, doi: https://doi.org/10.1007/s12596-024-02080-w.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiriyah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

[36] D. Bouras, M. Fellah, A. Mecif, R. Barillé, A. Obrosov, and M. Rasheed, "High photocatalytic capacity of porous ceramic-based powder doped with MgO," Journal of the Korean Ceramic Society, Oct. 2022, doi: https://doi.org/10.1007/s43207-022-00254-5.

[37] M. Rasheed, S. Shihab, O. Alabdali, A. Rashid, and T. Rashid, "Finding Roots of Nonlinear Equation for Optoelectronic Device," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012077, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012077.

[38] M. Rasheed, O. Alabdali, S. Shihab, A. Rashid, and T. Rashid, "On the Solution of Nonlinear Equation for Photovoltaic Cell Using New Iterative Algorithms," Journal of Physics: Conference Series, vol. 1999, no. 1, p. 012078, Sep. 2021, doi: https://doi.org/10.1088/1742-6596/1999/1/012078.

[39] Aasim Jasim Hussein, Mustafa Nuhad Al-Darraji, and M. Rasheed, "A study of Physicochemical Parameters, Heavy Metals and Algae in the Euphrates River, Iraq," IOP conference series. Earth and environmental science, vol. 1262, no. 2, pp. 022007–022007, Dec. 2023, doi: https://doi.org/10.1088/1755-1315/1262/2/022007.

[40] Selma, M. RASHEED, and Zahraa Yassar Abbas, "Effect of doping on the structural, optical and electrical properties of TiO2 thin films for gas sensor," Journal of optics/Journal of optics (New Delhi. Print), May 2024, doi: https://doi.org/10.1007/s12596-024-01913-y.

[41] M. Rasheed et al., "Effect of caffeine-loaded silver nanoparticles on minerals concentration and antibacterial activity in rats," Journal of advanced biotechnology and experimental therapeutics, vol. 6, no. 2, pp. 495–495, Jan. 2023, doi: https://doi.org/10.5455/jabet.2023.d144.

[42] Farouk BOUDOU, Abdelmadjid GUENDOUZI, A. BELKREDAR, and M. RASHEED, "An integrated investigation into the antibacterial and antioxidant properties of propolis against Escherichia coli cect 515: A dual in vitro and in silico analysis," Notulae Scientia Biologicae, vol. 16, no. 2, pp. 13837–13837, May 2024, doi: https://doi.org/10.55779/nsb16211837.

[43] H. K. Aity, E. Dhahri, and M. Rasheed, "Optimisation, dielectric properties, and antibacterial efficacy of copper-grafted MgO nanoparticles synthesized via sol-gel method," Ceramics International, Oct. 2024, doi: https://doi.org/10.1016/j.ceramint.2024.10.324.

[44] Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "The conjugate gradient approach to solve two dimensions linear elliptic boundary value equations as a prototype of the reaction diffusion system," Al-Salam journal for engineering and technology, vol. 3, no. 1, pp. 157–168, Jan. 2024, doi: https://doi.org/10.55145/ajest.2024.03.01.014.

[45] M. Enneffatia, M. Rasheed, B. Louatia, K. Guidaraa, S. Shihab, and R. Barillé, "Investigation of structural, morphology, optical properties and electrical transport conduction of Li0.25Na0.75CdVO4 compound," Journal of Physics: Conference Series, vol. 1795, no. 1, p. 012050, Mar. 2021, doi: https://doi.org/10.1088/1742-6596/1795/1/012050.

[46] M. Rasheed, M. N. Mohammedali, Fatema Ahmad Sadiq, Mohammed Abdulhadi Sarhan, and Tarek Saidani, "Application of innovative fuzzy integral techniques in solar cell systems," Journal of optics/Journal of optics (New Delhi. Print), Jun. 2024, doi: https://doi.org/10.1007/s12596-024-01928-5.

[47] Ahmed Shukur, "Application of Error Continuous Distribution in Analyzing Systematic Variability across Engineering Processes", Journal of Positive Sciences, Vol. 4, Issue: 1, pp: 47-54, (2024). doi: https://doi.org/10.52688/ASP58911.

[48] Ahmed Shukur, " Application of Error Function Continuous Distribution in Predictive Modeling and Quality Control", Journal of Positive Sciences, Vol. 4, Issue: 3, pp: 53-61, (2024). doi: https://doi.org/10.52688/ASP84163.

[49] Mohammed RASHEED, "Analyzing Applications and Properties of the Exponential Continuous Distribution in Reliability and Survival Analysis", Journal of Positive Sciences, Vol. 4, Issue: 5, pp: 71-79, (2023). doi: https://doi.org/10.52688/ASP30767.

[50] Mohammed RASHEED, "Modeling and Analysis of Extreme Events using Extreme Value Continuous Distribution", Journal of Positive Sciences, Vol. 4, Issue: 1, pp: 55-63, (2024). doi: https://doi.org/10.52688/ASP37713.

[51] Ahmed Shukur, "Sequential Event Modeling and Reliability Analysis using the Erlang Continuous Distribution", Journal of Positive Sciences, Vol. 3, Issue: 5, pp: 64-70, (2023). doi: https://doi.org/10.52688/ASP85431.

[52] A. Shukur, Ahmed Shawki Jaber, M. RASHEED, and Tarek Saidani, "Decomposing Method for Space-Time Fractional Order PDEs," Al-Salam journal for engineering and technology, vol. 3, no. 2, pp. 1–11, May 2024, doi: https://doi.org/10.55145/ajest.2024.03.02.01.

[53] Ahcen Keziz, Meand Heraiz, M. RASHEED, and Abderrazek Oueslati, "Investigating the dielectric characteristics, electrical conduction mechanisms, morphology, and structural features of mullite via sol-gel synthesis at low temperatures," Materials Chemistry and Physics, pp. 129757–129757, Jul. 2024, doi: https://doi.org/10.1016/j.matchemphys.2024.129757.

[54] A. Raghdi, Menad Heraiz, M. Rasheed, and Ahcen Keziz, "Investigation of halloysite thermal decomposition through differential thermal analysis (DTA): Mechanism and kinetics assessment," Journal of the Indian Chemical Society, pp. 101413–101413, Oct. 2024, doi: https://doi.org/10.1016/j.jics.2024.101413.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

[55] W. Saidi, Nasreddine Hfaidh, M. Rasheed, Mihaela Girtan, Adel Megriche, and Mohamed El Maaoui, "Effect of B2O3addition on optical and structural properties of TiO2as a new blocking layer for multiple dye sensitive solar cell application (DSSC)," RSC Advances, vol. 6, no. 73, pp. 68819–68826, Jan. 2016, doi: https://doi.org/10.1039/c6ra15060h.

[56] Habiba K. Aity, Muwafaq A. Hasan, Mohammed RASHEED, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Haider s. Mohammed, Mohammed H. Ali, Sammah Dammaka, Radhia Dhahri, Ahmed RASHID, Tarek Saidani, "Evaluating structural properties and antibacterial activity of MgxCu1-xO nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 5, pp: 9-19, (2024). doi: https://doi.org/10.52688/ASP72524.

[57] E. Kadri, M. Krichen, R. Mohammed, A. Zouari, and K. Khirouni, "Electrical transport mechanisms in amorphous silicon/crystalline silicon germanium heterojunction solar cell: impact of passivation layer in conversion efficiency," Optical and Quantum Electronics, vol. 48, no. 12, Nov. 2016, doi: https://doi.org/10.1007/s11082-016-0812-7.

[58] M. Rasheed, M. Nuhad Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "The numerical Calculations of Single-Diode Solar Cell Modeling Parameters," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012058, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012058.

[59] M. Rasheed, M. N. Al-Darraji, S. Shihab, A. Rashid, and T. Rashid, "Solar PV Modelling and Parameter Extraction Using Iterative Algorithms," Journal of Physics: Conference Series, vol. 1963, no. 1, p. 012059, Jul. 2021, doi: https://doi.org/10.1088/1742-6596/1963/1/012059.

[60] Habiba K. Aity, Ruqaya Shaker Mahmood, Muwafaq A. Hasan, Mohammed RASHEED, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Haider s. Mohammed, Mohammed H. Ali, Sammah Dammaka, Radhia Dhahri, Ahmed RASHID, Nareman Chakchouk, "Exploring the structural features and antimicrobial functionality of Mg0.95Cu0.05O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 5, pp: 30-40, (2024). doi: https://doi.org/10.52688/ASP57261.

[61] Habiba K. Aity, Rana A. Hadi, Ruqaya Shaker Mahmood, Mohammed RASHEED, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Nareman Chakchouk, "The relationship between optical characteristics and antibacterial performance of Mg0.97Cu0.03O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 5, pp: 30-40, (2024). doi: https://doi.org/10.52688/ASP33167.

[62] Habiba K. Aity, Kawther A. Alameri, Mohammed RASHEED, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Olfa Maalej, "The effect of structure on antibacterial performance of Mg0.94Cu0.06O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 1-11, (2024). doi: https://doi.org/10.52688/ASP441661.

[63] Habiba K. Aity, Esra A. Hashem, Mohammed RASHEED, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Olfa Maalej, "The role of optical properties in enhancing antimicrobial activity of Mg0.94Cu0.06O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 12-22, (2024). doi: https://doi.org/10.52688/ASP19241.

[64] Habiba K. Aity, Mohammed RASHEED, Kawther A. Alameri, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Marwa Enneffati, "Structural properties and bacterial inhibition capabilities of Mg0.91Cu0.09O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 23-33, (2024). doi: https://doi.org/10.52688/ASP28610.

[65] Habiba K. Aity, Mohammed RASHEED, Esra A. Hashem, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Marwa Enneffati, "Assessing Optical Behavior and Antibacterial Potency of Mg0.91Cu0.09O Nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 34-43, (2024). doi: https://doi.org/10.52688/ASP80838.

[66] Mohammed RASHEED, Ketam K. Khudair, Habiba K. Aity, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Taha Rashid, "The impact of optical characteristics on antibacterial properties of Mg0.99Fe0.01O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 54-63, (2024). doi: https://doi.org/10.52688/ASP75371.

[67] Mohammed RASHEED, Habiba K. Aity, Ketam K. Khudair, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Taha Rashid, "The influence of structural properties on antibacterial potential of Mg0.95Fe0.03O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 64-74, (2024). doi: https://doi.org/10.52688/ASP50513.

[68] Ketam K. Khudair, Habiba K. Aity, Ruqaya Shaker Mahmood, Mohammed RASHEED, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Taha Rashid, "Optical analysis and its impact on antibacterial performance of Mg0.97Fe0.03O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 75-86, (2024). doi: https://doi.org/10.52688/ASP88399.

[69] Rafal A. Obayed, Habiba K. Aity, Ruqaya Shaker Mahmood, Mohammed RASHEED, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Ahmed RASHID, Taha Rashid, "Investigating structural traits and their role in antibacterial properties of Mg0.94Fe0.06O nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 6, pp: 87-96, (2024). doi: https://doi.org/10.52688/ASP75220.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiriyah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

[70] Habiba K. Aity, Rana A. Hadi, Mohammed RASHEED, Ruqaya Shaker Mahmood, Farqad A. Rashid, Zahraa Abbas, Areej A. Hateef, Haider s. Mohammed, Mohammed H. Ali, Sammah Dammaka, Radhia Dhahri, Ahmed RASHID, Tarek Saidani, "Optical behavior and its role in the antimicrobial properties of MgxCu1-xO nanoparticles", Journal of Positive Sciences, Vol. 4, Issue: 5, pp: 20-29, (2024). doi: https://doi.org/10.52688/ASP66329.

[71] Areej Adnan Hateef, Essebti Dhahri, M. Rasheed, Habiba Kadhim, Z. Abbas, and N. Hassan, "Study of the influence concentration difference of copper in properties of cerium nanopowder," *Physics and Chemistry of Solid State*, vol. 25, no. 4, pp. 801–810, (2024), doi: https://doi.org/10.15330/pcss.25.4.801-810.

[72] A Jaber, Taha Rashid, Mohammed Rasheed, Ruqaya Shaker Mahmood, Olfa Maalej, Analysis of Cauchy Distribution and Its Applications, Journal of Positive Sciences, vol. 4, Issue 4, (2024). doi: https://doi.org/10.52688/ASP54542.

[73] Mohammed Rasheed, Ahmed Jabber, Mohammed Sarhan, Tarek Saidani, Mustafa Al-darraji, Two mathematical models for solving the photovoltaic cell's nonlinear formula, Journal of New Technology and Materials, vol. 13, Issue. 1, pp: 17-24 (2023).

[74] Taha Rashid, Muhammad bin Hamzah, MOHAMMED RASHEED, Ahmed Jaber, Mohammed Sarhan, Mustafa Aldaraji, Tarek Saidani, Image segmentation for animals in the wild using scilab software. Al-Salam Journal for Engineering and Technology, vol. 2, Issue 2, pp: 72-77. (2023). doi: https://doi.org/10.55145/ajest.2023.02.02.009.

[75] Aldaraji, M., Al-Azzawi, T., RASHEED, M., Sarhan, M., Jaber, A., Saidani, T., & Rashid, T. (2023). Investigating the effect of consuming soft drinks on the liver and kidney functions of a sample from the Iraqi community. *Al-Salam Journal for Engineering and Technology*, *2*(1), 103–108. doi: https://doi.org/10.55145/ajest.2023.01.01.0012.

[76] Abdul-jabbar, A., Rashid, T., RASHEED, M., Sarhan, M., Jaber, A., Aldaraji, M., Saidani, T., & Rashid, A. An NMOS transistor-based high-gain operational amplifier designed in 0.25-micron CMOS technology. *Al-Salam Journal for Engineering and Technology*, *2*(1), 123–133. (2023). doi: https://doi.org/10.55145/ajest.2023.01.01.015.

[77] RASHEED, M., Majeed, S., Alabdali, O., Abbood, M., & Ounis, T. (2023). Preparation and Characterization of Nickel Oxide for Antibacterial Properties: Nickel Oxide for Antibacterial Properties. *Al-Salam Journal for Engineering and Technology*, *2*(1), 117–122. doi: https://doi.org/10.55145/ajest.2023.01.01.0014.

[78] Jaber, A., Sarhan, M., Al-Darraji, M., RASHEED, M., Saidani, T., Rashid, T., & Rashid, A. (2023). A study of the physiological disorders of slimming drugs on people who abuse them. *Al-Salam Journal for Medical Science*, *2*(1), 21–27. doi: https://doi.org/10.55145/ajbms. 2023.1.1.004.

[79] Aldaraji, M., Al-Azzawi, T., RASHEED, M., Sarhan, M., Jaber, A., Saidani, T., & Rashid, T. (2023). Investigating the effect of consuming soft drinks on the liver and kidney functions of a sample from the Iraqi community. *Al-Salam Journal for Engineering and Technology*, *2*(1), 103–108. doi: https://doi.org/10.55145/ajest.2023.01.01.0012.

[80] Rashid, T., RASHEED, M., Sarhan, M. A. ., Jaber, A. S. ., Aldaraji, M. N. ., Saidani, T., & Rashid, A. . (2023). Folded Cascode Operational Amplifier Design Utilizing 0.25 µm CMOS Technology . *Al-Salam Journal for Engineering and Technology*, *2*(1), 76–84. doi: https://doi.org/10.55145/ajest.2023.01.01.009.

[81] Saidani T, Rasheed M, Alshalal I, Rashed AA, Sarhan MA, Barille R, Saidani T, Rasheed M, Alshalal I, Rashed AA, Sarhan MA. Characterization of thin ITO/Au/ITO sandwich films deposited on glass substrates using DC magnetron sputtering. doi: Res. Eng. Struct. Mater. vol. 10, no. 2: pp: 743-770. (2024). doi: https://doi.org/10.17515/resm2023.21ma0922rs.

[82] Rashid, T., RASHEED, M., Sarhan, M., Jaber, A., Aldaraji, M., Saidani, T., & Anwar Abdul-jabbar, A. (2022). A study of MOSFET Device for The characterization of ID-VG and ID-VD Curves. *Al-Salam Journal for Engineering and Technology*, *2*(1), 40–45. doi: https://doi.org/10.55145/ajest.2023.01.01.005.

[83] RASHEED, M., Rashid, T., bin Hamzah, M. N., Jaber, A. S., Sarhan, M. A., Aldaraji, M. N., Saidani, T., & Rashid, A. . (2023). Design A Combinational Circuit Consists of 10 Logic Gates Using Quartos. *Al-Salam Journal for Engineering and Technology*, *2*(1), 85–93. doi: https://doi.org/10.55145/ajest.2023.01.01.0010.

[84] Mohammed RASHEED, Mohammed Abdelhadi Sarhan, Ahmed Shawki Jaber, Dikra Bouras, Mustafa Nuhad Al-Darraji, Ahmed Rashid. (2022). Statistical Analysis of Copper Doped Titanium Dioxide. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *14*(3), Math Page 80–88. doi: https://doi.org/10.29304/jqcm.2022.14.3.1030.

[85] RASHEED, M., Sarhan, M. A., Jaber, A. S., Bouras, D., Al-Darraji, M. N., & Rashid, A. (2022). Statistical Analysis of Aluminum Doped Titanium Dioxide Using Solid State Method: Statistical Analysis. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *14*(3), Math Page 1–8. doi: https://doi.org/10.29304/jqcm.2022.14.3.955.

[86] RASHEED, M., Rashid, A., Rashid, T., Abed Hamed, S. H., & Sabri, A. A. (2021). Analysis of Non-Linear Device by means of Numerical Algorithms. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *13*(3), Math Page 79– 87. doi: https://doi.org/10.29304/jqcm.2021.13.3.845.

[87] RASHEED, M., Rashid, A., Rashid, T., Abed Hamed, S. H., & Manoochehr, N. T. (2021). Various Numerical Methods for Solving Nonlinear Equation. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *13*(3), Math Page 88– 97. doi: https://doi.org/10.29304/jqcm.2021.13.3.846.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq

[88] SHIHAB, S., RASHEED, M., Rashid, A., Rashid, T., Abed Hamed, S. H., & Manoochehr, N. T. (2021). An Accelerated Algorithm for Finding Roots of Nonlinear Equation. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *13*(2), Math Page 160– 168. doi: https://doi.org/10.29304/jqcm.2021.13.2.811.

[89] RASHEED, M., SHIHAB, S., RASHID, T., & Ounis, T. D. (2021). Determination of PV Model Parameters Using Bisection and Secant Methods. *Journal of Al-Qadisiyah for Computer Science and Mathematics*, *13*(1), Math Page 43– 54. doi: https://doi.org/10.29304/jqcm.2021.13.1.744.

[90] M. A. Sarhan, S. SHIHAB, and M. RASHEED, “A Novel Spectral Modified Pell Polynomials for Solving Singular Differential Equations”, *Al-Mustansiriyah J. Sci.*, vol. 32, no. 1, pp. 18–24, Feb. 2021, doi: 10.23851/mjs.v32i1.930.

[91] Abbas MM, Rasheed M. Investigation of structural, Mechanical, Thermal and Optical Properties of Cu Doped TiO2. Iraqi Journal of Physics. 2021 Mar 1;19(48):1-9. doi: https://doi.org/10.30723/ijp.v19i48.615.

[92] Hussein AM, Abdullah SA, RASHEED M, Zamel RS. Optical and electrical properties of glass/graphene oxide thin films. Iraqi Journal of Physics. 2020 Dec 1;18(47):73-83. https://doi.org/10.30723/ijp.v18i47.617.

[93] Mohammed Rasheed, Regis Barille, Development and characterization of single and multilayer thin films for optoelectronics application, Ph. D. Thesis University of Angers, (2017).

[94] Rasheed MS, Mohammed AN. Design of a laser based free space communication system. LAP LAMBERT Academic Publishing; 2012 Oct 20.

*Corresponding author
Faez N. Ghaffoori,
Department of Mathematics, College of Basic Education, University Mustansiryah, Baghdad,
e-mail: fng_2022@uomustansiriyah.edu.iq